Implications of Innovative Engineering Design on Closed Loop Supply Chain Coordination: Incentives for Input Material Reduction vs. Enhanced Recycling

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Motivated by interactions with a major player in the aerospace industry, we consider the relationship between a supplier of specialty material forgings and a buyer that manufactures fabricated airplane components by extensively machining down these forgings as per complex design specifications. Due to high material removal costs, the buyer prefers these forgings to be as similar in geometry and size to the final component as possible, i.e., near-net-shape. The supplier, by default, does not have the capabilities to deliver such near-net-shape forgings as per technological constraints, but can utilize costly effort and/or invest in the required technologies to achieve forging size reduction. By taking into account uncertainty regarding the correspondence between supplier's effort and resulting output (i.e., reduction in forging size), and potential information asymmetry issues due to supplier's private information regarding costly effort, we assess the implications of two innovative approaches our study firm considers for improving supply chain performance: (i) The buyer can partially subsidize the supplier to induce forging size reduction efforts, and/or (ii) the supply chain can facilitate a higher rate recycling of scrap material to reduce input material costs.

We find that the supplier's input reduction efforts and enhanced recycling across the supply chain interact in non-intuitive ways, thus inflicting a non-monotone effect on supply chain performance; in other words, an increased recycling rate may misalign incentives, and not reduce decentralization cost. We find that enhanced recycling deteriorates supply chain performance the most especially when the recycling rate is moderate, which suggests that our study firm should participate in a supply chain with either superb or insufficient recycling capabilities. Furthermore, when agency issues arise, we find that the buyer should subsidize supplier's forging size reduction efforts only when the recycling level is above a threshold, which helps improve supply chain performance. In contrast, when the recycling level is below the same threshold, as the buyer bets on the possibility on contracting with an efficient supplier, the buyer withholds any compensation at equilibrium, and thus, enhanced recycling yields a higher decentralization cost.

Key words: Closed loop supply chain, recycling, engineering design, input material reduction, contracting, coordination, agency theory.

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1. Introduction

Understanding the implications of product design and architecture on supply chain performance has received considerable attention by researchers and practitioners in operations and supply chain management as well as new product development. (See, for example, Ulrich, 1995; Novak and Eppinger, 2001; Ulku and Schmidt, 2011; Raz et al., 2013.) The argument that innovative product designs can enhance the efficiency of the subsequent production activities (such as assembly) dates back to the origination of design-for-manufacture and lean manufacturing principles (Knight and Boothroyd, 2005). As supply chains have continued to become more vertically disintegrated during subsequent decades, it has become more difficult for manufacturers to influence component suppliers' engineering design choices and process technology capabilities, thus resulting in various inefficiencies within the supply chain; for example, input material overuse, unnecessary machining and material removal, and ensuing excessive scrap. As a result, there has been an increased focus on reducing and recycling of materials for lean and environmentally conscious manufacturing.

During our interactions with a major player in the aerospace industry, we encountered a problem setting that highlights the aforementioned inefficiencies and supplier incentive issues, where innovations regarding product/engineering design and manufacturing technologies *also* have significant recycling implications. The specific problem at our study firm presents itself in a setting where a buyer procures specialty material forgings for large structural components that are intricate in geometry. The buyer faces excessive machining and material removal costs, because the product design specifications drive the standard size forgings provided by its supplier to be up to 30-40 times the size of the final weight of the part. (The ratio of the forging weight to the final component weight is referred to as the "buy-to-fly" ratio in this industry.) The supplier in this case does not typically have an incentive to invest in innovative forging capabilities that can offer more "near-net-shape" forgings (with low buy-to-fly ratio) for these parts. Recent research in supply chain management has offered mechanism design to resolve the ensuing incentive misalignment issues, and highlighted manufacturers' processing cost benefits of incentivizing suppliers' capabilities to support innovative product/engineering designs. However, not only it is difficult to design contracts that coordinate supply chain members' optimal actions, but also such contracts, albeit effective in theory, are difficult to administer and/or verify in practice due to potential information asymmetry issues. In this study, we investigate *whether facilitating scrap material reuse via recycling can help coordinate a bilateral supply chain as an alternative to the traditional subsidies manufacturers offer to suppliers for investing in innovative technologies that permit input reduction.*

Specific research questions of interest for this study are the following: How do the two alternatives under consideration, i.e., input reduction incentives via contracting versus enhanced scrap material recycling, affect the supplier's and the buyer's decisions? What terms are optimal for the contract the buyer should offer at equilibrium, and what effort level on the supplier's part does that contract induce? What are the effects of recycling rate improvements on various supply chain inefficiencies? Can the aforementioned (recycling) improvements have a significant enough impact to induce supply chain coordination, while making the supply chain more sustainable in terms of virgin material use as input? The last two questions, especially, are of significant interest to our study firm, which is the buyer in our setting, as the firm recently formed a joint venture with the forging supplier located near the virgin material source to facilitate a higher rate recycling of scrap material generated during the machining process at the study firm.

Whereas our study firm operates in the aerospace industry where long-term relations with suppliers are common due to strict quality requirements, thus making supplier capabilities known to the buyer, such transparency between the buyer and the supplier is not standard in other industries that use custom forgings. Even in the aerospace industry, when buyers look to contract with new suppliers offering different technological capabilities, the buyers may not accurately predict supplier capabilities and/or the suppliers' cost and effort levels to utilize certain technologies. With such information asymmetry issues in mind, we also consider the aforementioned research objectives in an adverse selection setting where the supplier's type—defined as its cost to achieve a given level of input reduction—is unknown to the buyer.

It is worth noting that, although we describe our setting using the terminology that is consistent with our study firm's context, our analysis and findings would apply to any bilateral closed loop supply chain setting, where the supplier's effort and the ensuing product design affect the buyer's processing costs, and the virgin-to-recycled material ratio (influenced by the scrap recycling rate) affects the supplier's costs. Similar challenges are apparent in the metals manufacturing industry wherein buyer firms desire input reduction via net-shaped or near-net-shaped forged components, and yet have to contend with forging sizes far from desired, resulting in excessive machining waste. Custom forging in North America accounts for over 6 billion dollars in sales annually, which is carried out by about 250 forging companies in approximately 300 plants across the United States, Canada, and Mexico.[1]

Seeking explanations to the aforementioned research questions, we consider the relationship

[1] https://www.forging.org/about#how-big.

between a supplier of specialty material forgings and a buyer that manufactures airplane components with intricate geometry by extensively machining down these forgings as per design specifications. The material removal costs of the buyer increases with the processing time to remove excess material, (which, in our study firm's setting, can be as long as 24 hours) and thus the buyer prefers these forgings to be as similar in geometry and size to the final component as possible, i.e., near-net-shape. The supplier, by default, does not have the capabilities to deliver such near-net-shape forgings as per its technological constraints, but can utilize costly effort and/or invest in the required technologies to achieve forging size reduction. We first assume that both the buyer and supplier know the supplier's aforementioned "forging size reduction" costs, and then relax this assumption to incorporate information asymmetry considerations. In the latter case, we permit two supplier types: an inefficient type, and an efficient type that can achieve the same forging size reduction level at a lower cost. In both information scenarios, the buyer can induce this costly effort on the supplier's part by offering monetary incentives, which would partially compensate for the supplier's effort and costs. Alternatively, the buyer can facilitate recycling excess scrap material, a by-product of the material removal process, back to the forging supplier, which reduces the input virgin material requirements, thus yielding material cost savings for the supplier.

In the decentralized setting, the buyer moves first by offering a contract (or, possibly, a menu of contracts in the asymmetric information case) that pays the supplier a two-part-tariff, i.e., an upfront payment and a reward proportional to the anticipated forging size reduction the supplier will achieve. Then the supplier responds by exerting costly effort while taking into account his rewards and material costs, where the latter depend on the virgin-to-recycled material ratio. We permit the supplier's marginal effort cost per unit reduction in forging size to increase to reflect practice in the aerospace industry, where technological improvements yield marginally decreasing benefits. We assume that the mapping between each supplier type's effort level and the resulting forging is known by both parties in expectation; however, the actual forging size that results varies to reflect the uncertainty regarding the investment in the requisite technology and the resulting output. In effect, the supplier is paid *ex ante* according to the aforementioned expectation, and cannot be penalized *ex post*, which we reflect in our model by assuming that the supplier has "limited liability." In other words, we restrict the transfer payment implied by the contract to non-negative values. Once the uncertainty regarding the output (i.e., the resulting forging size) is resolved, the buyer incurs material removal costs as a function of how near-net-shape the forging procured from the supplier is.

We first analyze a benchmark setting where the buyer and the supplier act together, i.e., a centralized solution. In this case, the contract terms are irrelevant as transfer payments between the buyer and the supplier remain within the supply chain. Similarly, the information asymmetry issues are inconsequential. We find that the optimal effort the supplier exerts at first-best is non-increasing in the recycling rate, thus highlighting the fact that near-net-shape forgings and recycling at a higher rate are *substitutes* that generate similar benefits for the supply chain; the former yields material cost savings via input reduction, whereas the latter achieves the same by reusing the less expensive input form.

When the supply chain members act individually, an incentive misalignment issue arises due to the aforementioned substitution effects. In effect, the supplier does not exert the same effort for a given recycling rate when compared to the first-best effort level. More specifically, whereas a higher recycling rate yields material cost savings for the supplier, at the same time, it discourages the supplier to exert costly effort and pass the savings to the buyer. As such, at higher recycling rates, the buyer has to offer a higher payment to encourage forging size reductions. Yet, he must do so without an upfront payment when the supplier's type is known. This is because, without the ability to punish the supplier for less than expected forging size reductions due to the limited liability of the supplier, the buyer should only promise a reward for efforts towards an anticipated forging size reduction.

When the supplier's type is unknown to the buyer, we find that either a pooling or a separating equilibrium may result. The former sustains when it is optimal for both types to exert positive effort at the first-best. In this scenario, the buyer cannot utilize a menu of contracts to properly incentivize the supplier and distinguish its type. Consequently, the efficient type would respond by exerting more effort than the inefficient type would. When a separating equilibrium sustains, the buyer can distinguish types by devising different contracts for each type, where the efficient type is offered both an upfront payment and a reward for input reduction, whereas the inefficient type's reward is only performance based—albeit at a rate higher than the reward rate for the efficient type.

Regarding the value of improved recycling capabilities, we study the changes in supply chain profits as recycling rate improves. Our focus on supply chain profits reflects the practice at our study firm and its supplier partner; in other words, we assume that as supply chain partners invest together in better recycling capabilities, they should find a way to equitably allocate the ensuing savings such that no single party pockets all the economic value created. We find that the supply chain efficiency implications of enhanced recycling are non-intuitive, in other words, *better recycling* *can increase or decrease decentralization cost.* This finding has significant managerial implications for the metals manufacturing industry for custom forgings, as it issues caution regarding costly investments to improve recycling capabilities. We find that, when the recycling rate is low or high, enhanced recycling reduces decentralization cost by preventing the supplier from over-investing and under-investing, respectively, in forging size reduction. In contrast, at moderate-to-medium recycling levels, enhanced recycling elevates decentralization cost due to two reasons: The recycling rate is not low enough to encourage supplier's input reduction effort, while, at the same time, it is not high enough to encourage the buyer to address the incentive misalignment issue by subsidizing the supplier's input reduction cost.

We also characterize the conditions under which the marginal benefit from better recycling for the supply chain when the buyer and the supplier act individually can exceed the marginal benefit from better recycling at the first-best. In such cases, the improvements can be so significant that *the supply chain can fully eliminate the ensuing decentralization cost*, and mitigate the adverse effects of the misalignment issues. Moreover, we find that the aforementioned recycling benefits can prevail even when there is information asymmetry issues, and a higher recycling rate can lower the decentralization cost even when a pooling equilibrium results. Consequently, our findings highlight that the partnerships such as the one formed by our study firm and its supplier to enhance recycling capabilities can complement more traditional supply chain coordination mechanisms such as effort inducing subsidy contracts. With that being said, as potential suppliers' input reduction capabilities (and costs) become more asymmetric, enhanced recycling can increase decentralization cost, unless the recycling rate is above a threshold.

The rest of the paper is organized as follows. We survey the extant literature in §2. Then, in §3, we describe model basics. We analyze both the first-best and the principle-agent settings in §4, where we assume the buyer knows the supplier's parameters, while permitting stochastic output levels resulting from the supplier's effort. In this section, we also investigate how better recycling capabilities influence supply chain inefficiencies due to decentralization. In §5, we extend our analysis to account for information asymmetry considerations, where the supplier's cost-to-effort mapping is the supplier's private information. Section 6 concludes. We relegate all technical proofs to the Appendix.

2. Related Literature

The problem we study in this paper manifests itself in a closed loop supply chain (CLSC) setting where the product architecture and the supplier's manufacturing capabilities influence both forward (virgin) and reverse (recycled) material flows, and thus affect all involved parties' costs and payoffs. As such, our work addresses a gap in existing literature, which Ferguson and Souza (2010) highlight as *the need to bring product/engineering design for sustainability issues in a rigorous manner into the closed loop supply chain research.*" In addition, as advocated by Guide and Van Wassenhove (2009), our research is strongly rooted in CLSC practice, as all facts regarding our problem context are established via direct engagements with product design and engineering teams at our study firm, which is a leading player in the aerospace industry.

Our research relates to three literature streams, namely research on CLSC with particular focus on recycling, product design literature with sustainability considerations, and supply chain coordination literature that utilizes a principle-agent framework. Each of these research streams have been well-established and comprises hundreds of published and working papers, and thus we will limit this section on related literature to recent surveys, when they are available, and highlight in detail only a few papers that closely relate to our modeling choices.

For a comprehensive survey of the CLSC literature, we refer readers to reviews by Atasu et al. (2008) and Guide and Van Wassenhove (2009), which provide classifications of CLSC research with both supply- and demand-side considerations. Also of note is the survey by Akcali et al. (2009), which highlights models and solution approaches for CLSC network design problems, including those that focus on reverse product flows by assessing the cost implications of incorporating recycled materials and/or reused products/components in remanufacturing activities. Our study also focus on reverse scrap flows generated by the buyer's material removal process and the corresponding cost implications for both the buyer and the supplier, yet, it is distinctly different from prior work as we relate product architecture and supplier's manufacturing capability enhancement efforts to CLSC material flow dynamics. Recently, Aydinliyim and Murthy (2016) considered a similar setting in terms of scrap material flows, yet, in their model, the supply side is exogenous, as the supplier does not engage in any investment to improve manufacturing capability to influence forging weight. Instead, they focused on the implications of the buyers component design choices (i.e., integral versus modular architecture) on the ensuing competition between incumbent and new supplier.

There is broad literature on product design that focus on operational issues (Ulrich, 1995; Novak and Eppinger, 2001; Ramdas et al., 2003; Ramachandran and Krishnan, 2008; Krishnan and Ramachandran, 2011). The interface of product design with sustainability has recently been exploited in a number of papers in operations management (OM) literature, which attempt to establish the link between product design decisions and reuse and recycling implications in CLSC; Ferguson and Souza (2010) survey the related literature until 2010. More recently, Ulku and

Schmidt (2011), and Agrawal and Ulku (2012) investigate how modularity affects sustainable product development and reuse. Galbreth et al. (2012) study how product innovation rate influence including of reused products in a product line. Raz et al. (2013) study how product design innovations affect product cost and demand as well as the environment. In our paper, we also consider innovation and the ensuing recyling/reuse implications, yet our focus is how innovation in supplier's manufacturing capabilities, which yields more near-net-shape forgings to be processed by the buyer, influences the buyer's cost and ability to recycle. The amount of recycling affects the supplier's cost as well, highlighting the closed loop nature of the buyer-supplier relationship we particularity focus on.

Our work also differs from the aforementioned CLSC and product design and sustainability literature in terms of the principle-agent (PA) framework we employ to study the buyer-supplier relationship we observed at our study firm's operations. (For a comprehensive review of the use of this framework in economics, we refer the reader to Bolton and Dewatripont, 2005.) PA models, also referred to as agency theory, has also been widely used in OM, particularly when one desired to study contractual relationships among supply chain members; Cachon (2003) reviews this OM literature prior to 2003. More recently, Plambeck and Taylor (2006), Lutze and Ozer (2008), Kim et al. (2007), and Kim and Netessine (2013) employ agency theory to study various supply chain settings with contracting and incentive alignment issues. Among these recent papers, Kim et al. (2007) is of note, as the authors also consider output uncertainty that is linked to the supplier's effort in expectation in addition to adverse selection. (In our setting, the supplier's forging-size-reduction efforts yield uncertain output, whereas the adverse selection results from the supplier's private input-reduction cost.) With that being said, the similarities between our work and all aforementioned papers in this literature end in our use of agency theory; our work distinctly focuses on focuses on the interplay between input material reduction via enhanced supplier process capability and CLSC considerations, wherein there exists a trade-off between input reduction and improved recycling.

3. Model

We consider a bilateral supply chain with one supplier and one buyer. The buyer procures specialty material (e.g., titanium) forgings from the supplier, and performs further material removal to produce the end-product (e.g., a specific airplane component) as dictated by design specifications. Without loss of generality, we focus on one unit weight of buyer's output, which is also consistent with our study firm's setting wherein the associated costs and rewards are measured in units proportional to specialty material weights in input/output forms. More specifically, if the buyer receives a forging that weighs x to obtain one unit weight of output, the material removal process yields $x-1$ unit weights of scrap material, which the buyer salvages via a third-party. (We will henceforth use "unit weights" and "units" interchangeably.) We assume fraction β of the scrap the buyer generates can be recycled back to the supplier's forging production to be reused as input, and thus the supplier's virgin input requirement is $x-\beta(x-1)$ units. We permit β to take values within the unit interval to account for various reverse material flow efficiency scenarios, where $\beta<1$ implies, for example, material leakages in the closed-loop supply chain, or possible technological constraints on maximum scrap material use. We highlight these material flow dynamics as well as the associated monetary flows in Figure 1.

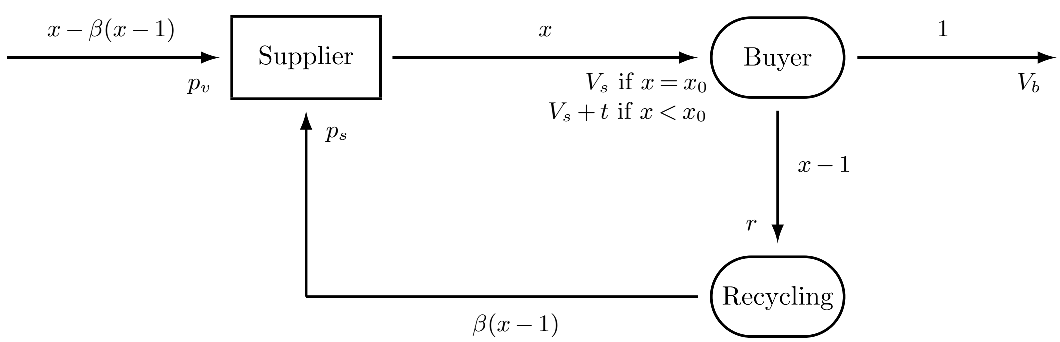

Figure 1 **Material (on the arrows) and monetary flows (at the tip of the arrows) that yield 1 unit of output when the buyer removes $x-1$ units of scrap from a forging that weighs x units. (In our study firm's context, this implies a buy-to-fly ratio of x-to-1.) The requisite amount of input material for the supplier is $x-\beta(x-1)$ units in virgin form and $\beta(x-1)$ units in scrap form. All payments are made by the party at the tip of the arrows to the party at the tail of the arrows.**

The forging size to end-product ratio is x_0-to-1 for the status-quo design the supplier provides to the buyer, thus necessitating x_0-1 units of material removal. As the buyer's material removal cost increases with the processing time to remove excess material, the buyer welcomes forgings that are as similar in geometry and size to the final component as possible, i.e., near-net-shape. Specifically, the buyer incurs material removal cost $M(x)$ $(1<x\le x_0)$ to perform material removal on a forging that weighs x times as much as the end-product. Design and processing engineers in our study firm suggest this cost to be convex increasing for operations that require extensive material removal, and thus we adopt function $M(x)\triangleq m_1x^2+m_2x$ to model the buyer's material removal cost. The supplier, by default, does not have the capabilities to deliver such near-net-shape forgings (i.e., $x<x_0$) as per technological constraints, but can utilize costly effort $e>0$ to achieve forging size reduction. (We illustrate the resulting forging size reduction in Figure 2.) We permit the supplier's marginal effort cost per unit reduction in forging size to increase according to function $K(e) \triangleq k_1 e^2 + k_2 e$ to reflect the forging suppliers' state of the art in the aerospace industry, wherein technological improvements yield marginally decreasing benefits. We assume that the

Figure 2 **(a) The status-quo forging with weight x_0 for a component with weight 1, i.e., buy-to-fly ratio of x_0-to-1, and (b) an improved forging with weight x for the same component, i.e., buy-to-fly ratio of x-to-1. In both panels, the light-colored area is the final component, and the dark-colored area is the scrap material the buyer will generate after machining, i.e., $x_0 - 1$ and $x - 1$ for the status-quo and improved designs, respectively.**

mapping between the supplier's effort level and the corresponding forging size reduction is known by both parties in expectation; however, the actual forging size that results varies due to uncertainty regarding the investment in the requisite technology and the resulting output. Mathematically, supplier's effort e yields forging size X satisfying

$$X = x_0 - e + \zeta. \tag{1}$$

Forging size X is a random variable as ζ is stochastic with mean 0 and standard deviation $\sigma > 0$ for $e > 0$. (We assume $\sigma \to 0$ as $e \to 0$, because if the supplier does not exert any effort the resulting forging will reflect the status-quo design, i.e., $X = x_0$ with probability 1 when $e = 0$.) The buyer compensates the supplier's effort *ex ante* according to his expectation for how near-net-shape the resulting forging would be by paying the supplier the transfer

$$t(X, \omega, \alpha) = \omega + \alpha(x_o - X). \tag{2}$$

This contract functions as a two-part tariff, where ω denotes a fixed up-front payment, and α is a reward for unit forging size reduction. We refer to the parameter α as the *compensation rate* for ease of explanation. Once the uncertainty regarding the output (i.e., the resulting forging size) is resolved, the buyer incurs material removal costs $M(x)$ *ex post*, where x is a realization of random variable X. Evidently, the resulting forging size may exceed the buyer's expectation for a given effort level, i.e, it is possible that $x > \mathbb{E}[X]$, in which case the buyer may consider a penalty. We do not permit such a penalty by restricting the transfer to non-negative values, i.e., $t(X,\cdot,\cdot) \geq 0$. In other words, we assume that the supplier has "limited liability."

We assume that the buyer's reward for satisfying unit demand is $V_b > 0$. The buyer also makes unit revenue $r > 0$ for recycling scrap that the material removal process yields. As the amount of material to be removed and scrapped depends on the stochastic forging size, the associated material removal cost $M(X)$ and the linear scrap revenue $r(X-1)$ are also stochastic. Also taking into account the forging size reduction reward the buyer pays as per equation (2), the buyer's payoff equals

$$\Pi_b(X,e) = V_b + r(X-1) - M(X) - t(X,\omega,\alpha), \tag{3}$$

where we assume the buyer's reservation utility is positive, i.e., $\mathbb{E}_X\left[\Pi_b(X,0)\right] > 0$. In other words, the buyer enjoys a profit by processing the status-quo forging with size x_0.

We assume that the supplier earns reward $V_s > 0$ to deliver a status-quo forging with size x_0, which is sufficient to cover its material cost of $p_v\big(x_0 - \beta(x_0-1)\big)$ for the requisite virgin material plus $p_s\big(\beta(x_0-1)\big)$ for the recycled scrap, where we denote by p_v and p_s the unit specialty material cost in virgin and recyled/scrap forms, respectively. If the supplier exerts costly effort e to achieve forging size reduction, it incurs effort cost $K(e)$, yet its reward increases as per transfer (2). Consequently, the supplier's stochastic payoff equals

$$\Pi_s(X,e) = V_s - \beta\Delta_p - \bar{p}(\beta)X - K(e) + t(X,\omega,\alpha), \tag{4}$$

where $\Delta_p \triangleq p_v - p_s > 0$ is the virgin-to-scrap price differential, and $\bar{p}(\beta) \triangleq (1-\beta)p_v + \beta p_s$ is the average material price given the effective recycling rate β.

4. Analysis: The Symmetric Information Case

In this section, we assume that the supplier's effort-to-output mapping is known by both parties in expectation; and thus, there are no information asymmetry considerations. Therefore, our analysis in this section applies to settings wherein the supplier and the buyer have an established relationship and/or they are related companies, so that critical cost parameters and technological capabilities are known to both. (An example setting would be the relationship between our study firm and its supplier, where the supplier is a joint venture of the buyer and the virgin specialty material provider.) In this setting, the only source of uncertainty that remains is regarding the actual forging size, i.e., realization x of the random variable X, the supplier can deliver for a given effort level e. We will first analyze a benchmark setting where the buyer and the supplier act together, i.e., the first-best solution. Then, we will analyze the setting where the buyer moves first by offering a transfer to incentivize the buyer to exert costly effort to achieve forging size reduction, and then the supplier responds. In both the centralized and the decentralized scenarios, we assume both parties are risk-neutral utility maximizers.

4.1. The First-Best Solution

In this subsection, we analyze a centralized decision maker's optimization problem with the objective to maximize the sum of the buyer's and the supplier's profits. As the transfer (2) remains within the supply chain, the only decision is to the supplier's forging size reduction effort. We denote by $e^{fb}(\beta)$ (e^{fb} in short) the supplier's optimal effort at the first-best solution, i.e.,

$$e^{fb} \triangleq \arg\max_{e \geq 0} \mathbb{E}_X \left[\Pi_s(X,e) + \Pi_b(X,e)\right].$$

It is easy to verify that the supply chain's objective is concave as material removal cost $M(\cdot)$ and effort cost $K(\cdot)$ are convex functions of their respective arguments, and thus convex optimization theory dictates that e^{fb} either satisfies a first-order-condition (FOC) or is a boundary solution. We further assume that $e^{fb} \geq x_0 - 1$ is not possible as technological constraints prohibit an "exact-net-shape" (i.e., $x = 1$) forging. We characterize the first-best solution in the next proposition.

PROPOSITION 1. *The first-best effort e^{fb} satisfies*

$$e^{fb} = \max\left\{\frac{2m_1x_0 + m_2 - k_2 + \bar{p}(\beta) - r}{2(m_1 + k_1)}, 0\right\} \tag{5}$$

for any given β, and is non-increasing in β.

Proposition 1 highlights that the bilateral supply chain we consider reaches its maximum performance when the supplier's optimal effort strikes a balance among various factors that relate to input reduction spending and material costs on the supplier's part, material removal costs and scrap material revenues on the buyer's part, and the status-quo input-to-output ratio x_0. For the status-quo design to remain optimal for the supply chain, i.e., expression (5) returning 0, the status-quo design should already be relatively near-net-shape (i.e., low x_0), and the average material price, and the buyer's material removal costs should be small relative to the supplier's effort cost and the buyer's salvage revenue for scrap. Proposition 1 also highlights that enhanced recycling capabilities and the supplier's input reduction efforts function as substitutes, i.e., the supply chain would settle for less near-net-shape input if it can recycle the by-product output with better yield. In the context of our study firm, who recently formed a joint venture with the forging supplier to facilitate a higher rate recycling of scrap material, this finding issues caution to buyers who consider recycling investments, because the resulting material savings may disincentivize the supplier's input reduction effort.

4.2. The Decentralized Supply Chain

We first establish a sequence of events for the setting wherein the buyer and the supplier act individually. Reflecting the practice at our study firm, we assume the buyer moves first by offering contract (2) to the supplier, which maps the supplier's effort to a two-part tariff only in expectation, as it is possible that $x \neq \mathbb{E}[X]$. Next, given the contract's upfront payment ω and its compensation rate α, the supplier commits to exerting costly effort e. The buyer then pays the supplier $\omega + \alpha(x_0 - X)$ in addition to base price V_s. Lastly, the supplier exerts effort at the pre-committed level, which yields a forging with size x, and the involved parties incur/earn other related costs/rewards.

We employ backward induction to characterize the players' equilibrium decisions; ω^* and α^* for the buyer, and supplier's best-response effort $e^{br}(\omega, \alpha, \beta)$ to the contract the buyer offers. For brevity, we denote the supplier's optimal effort by e^{br}, and first solve the supplier's problem of finding

$$e^{br} \triangleq \arg\max_{e \geq 0} \mathbb{E}_X \left[\Pi_s(X, e)\right].$$

when offered a contract with parameters ω and α. Similar to the aforementioned optimization to find the first-best, we use the convexity of the supplier's effort cost and the linear transfer to conclude that the optimal decision must either satisfy the FOC or is an extreme value. Consequently, the next proposition characterizes the supplier's optimal effort.

Proposition 2. *The supplier's optimal effort e^{br} satisfies*

$$e^{br} = \max\left\{\frac{\alpha - k_2 + \bar{p}(\beta)}{2k_1}, 0\right\} \tag{6}$$

for any given α and β, and does not depend on ω. Furthermore, e^{br} is non-decreasing in α and non-increasing in β.

Similar to e^{fb}, the supplier's optimal effort e^{br} decreases with the effort cost and increases with the average material price. However, as the supplier acts independently, the optimal effort does not depend on machine removal cost, thus necessitating an incentive by the buyer to induce input reduction efforts on the supplier's part. Evidently, this incentive should not comprise a fixed payment by the buyer, as equation (6) is independent of ω. We thus anticipate that $\omega^* \leq 0$ must hold at equilibrium, and the buyer should use a reward-based incentive by compensating the supplier's effort cost at a rate of α per unit input reduction. Consequently, how e^{br} compares with e^{fb} depends on the magnitude of the buyer's choice of α. If this compensation rate did not exceed $k_2 - \bar{p}(\beta)$, then the supplier would not exert any effort yielding the status-quo design x_0. In that case, output uncertainty would be inconsequential, and all parties' payoffs would remain at their corresponding reservation levels.

For cases where the supplier exerts optimal positive effort e^{br}, output (i.e., forging size reduction) uncertainty is relevant to the buyer's optimal contract choice. In such scenarios, it is conceivable that the buyer considers a high compensation rate (i.e., a relatively high α), while issuing the supplier a penalty (i.e., $\omega < 0$) for forgings that may be larger in size than expected for a given effort level (i.e., when $x > \mathbb{E}[X]$). Extant literature on contracting (such as Sappington, 1983 and Oyer, 2000) argues otherwise by stating that the effort-inducing party should not be held accountable disproportionately when a principal contracts with an agent *ex ante* whilst facing output uncertainty; in other words, the supplier in our setting must have limited liability. Mathematically, this argument constraints contract parameters to non-negative values, yielding the following corollary:

Corollary 1. *The limited liability assumption yields* $\omega^* = 0$.

Consequently, the buyer's optimization problem requires finding

$$\alpha^* \triangleq \arg \max_{\alpha \geq \left(k_2 - \bar{p}(\beta)\right)^+} \mathbb{E}_X\left[\Pi_b(X, e^{br})\right].$$

The quadratic function forms we employed to represent the supplier's effort cost and the buyer's machine removal cost ensures that the buyer's expected profit exhibits a unique maximum,[2] Furthermore, as the supplier's compensation V_s for delivering the status-quo forging with size x_0 already guarantees a positive surplus, any transfer that satisfies the limited liability assumption also ensures that the supplier's participation constraint is met. Consequently, it suffices to consider the buyer's unconstrained optimization problem, for which the next proposition characterizes an optimal solution.

Proposition 3. *The buyer's optimal compensation rate is*

$$\alpha^* = \max\left\{\alpha_1(\beta), \alpha_2(\beta)\right\} \tag{7}$$

for a given recycling rate β, *where*

$$\alpha_1(\beta) = k_2 - \bar{p}(\beta) + \tfrac{k_1}{2k_1 + m_1}\left(2m_1x_0 + m_2 - r + \bar{p}(\beta) - k_2\right) \quad and \quad \alpha_2(\beta) = \left(k_2 - \bar{p}(\beta)\right)^+,$$

[2] A less strict sufficient condition ensuring that the objective functions we considered thus far would support a unique maximum is $\frac{d^2M(x)}{dx^2} + e\frac{d^3K(e)}{de^3} > 0$ for all $x < x_0$ and $e \geq 0$, which would render the quadratic cost function assumptions moot. However, attaining closed-form expressions of the involved parties' equilibrium decisions is not possible sans the assumption of a specific form for these cost functions.

and α^* is non-decreasing in β. Consequently, optimal contract $t^*(X, \omega^*, \alpha^*)$ induces

$$e^* = \begin{cases} \left(\frac{2m_1x_0+m_2-r+\bar{p}(\beta)-k_2}{2(2k_1+m_1)}\right)^+ & \text{if } \alpha^* > 0, \\ \left(\frac{\bar{p}(\beta)-k_2}{2k_1}\right)^+ & \text{if } \alpha^* = 0. \end{cases} \tag{8}$$

Furthermore, e^* is non-increasing in β.

Proposition 3 highlights various effects regarding the optimal compensation rate and the supplier's optimal that bonus induces. Intuitively, α^* (when positive) increases with the buyer's material removal costs as well as the status-quo design's material requirement, as higher values for these parameters necessitates that the buyer incentivize supplier's input reduction efforts more aggressively. In addition, a positive compensation rate increases with the supplier's input reduction costs, as such higher costs make it more difficult for the supplier to undertake input reduction activities.

Less obviously, Proposition 3 also highlights that a contract with a higher compensation rate may not induce more input reduction efforts on the supplier's part. For example, a higher recycling rate in the supply chain reduces input costs, thus making the supplier more reluctant to engage in input reduction. In that case, the buyer should respond by increasing its incentive for the supplier by increasing the compensation rate. However, in equilibrium, the supplier induces less effort. As a result, input reduction and enhanced recycling function as substitutes in this setting, which yield incentive misalignment issues for the supplier.

Moreover, it is conceivable that the buyer chooses not to compensate the supplier at all by setting $\alpha^* = 0$, as evidenced by equation (7) possibly returning 0. More strikingly, even when α^* is positive, it may not be large enough to induce any input reduction effort by the supplier; see equation (8).

If, on the other hand, the recycling rate in the supply chain is low, the supplier may not need any incentives from the buyer to engage in input reduction. As the supplier's costs are partially driven by input material costs, a low β may yield a high enough average material cost $\bar{p}(\beta)$ that the buyer would exert a positive input reduction effort even when the buyer sets the compensation rate at 0.

4.3. Coordination Issues, and the Impact of Recycling

As evidenced by the discussion succeeding Proposition 3, the equilibrium that results from the buyer's contract choice and the supplier's response in input reduction effort is largely influenced by the level of recycling in the supply chain. These dynamics are likely to create incentive misalignment issues, which, in turn, may yield supply chain inefficiencies in the form of decentralization cost. In this subsection, we study such inefficiencies by comparing the equilibrium prescribed by Proposition 3 and the first-best solution; see equation (5).

Proposition 4. *Define $\underline{\beta}$ as*

$$\underline{\beta} \triangleq \max\Big\{\frac{p_v - k_2}{\Delta_p} - \Big(\frac{(2m_1x_0 + m_2 - r)k_1}{m_1\Delta_p}\Big)^+, 0\Big\}.$$

Then, we have the following statements:

(i) If $\alpha^* > 0$, then $e^{fb} \geq e^*$.

(ii) If $\alpha^* = 0$, then $e^{fb} \geq e^*$ if and only if $\beta \geq \underline{\beta}$.

Proposition 4 shows that when the buyer tries to incentivize input reduction at equilibrium, the optimal compensation rate $\alpha^* > 0$ is not sufficient to induce first-best effort on the supplier's part. It is even possible that when it is optimal for the buyer to not incentivize the supplier, i.e., $\alpha^* = 0$, the supplier may be satisfied with the material cost savings for input material due to a high recycling rate in the supply chain, and choose not to employ first-best effort. On the contrary, when the recycling rate remains below threshold level $\underline{\beta}$, the average input material cost becomes high enough that the supplier tries to reduce input material by exerting an input reduction effort. In the latter case, it is possible that the supplier's equilibrium effort not only is positive (despite no compensation from the buyer), but also exceeds the first-best effort.

Evidently, for scenarios where e^* and e^{fb} differ, the supplier-buyer relationship chain we consider cannot reach its profit generating potential, and decentralization cost ensues. Denoting the supply chain's first-best and equilibrium profits, respectively, by $\Pi^{fb}(\beta) \triangleq \mathbb{E}\big[\Pi_s(X, e^{fb}) + \Pi_b(X|e^{fb}\big]$ and $\Pi^*(\beta) \triangleq \mathbb{E}\big[\Pi_s(X, e^*) + \Pi_b(X|e^*)\big]$, we can express decentralization cost $\Delta(\beta)$ as

$$\Delta(\beta) = 1 - \frac{\Pi^*(\beta)}{\Pi^{fb}(\beta)}. \tag{9}$$

Note in equation (9) our emphasis on the decentralization cost implications of the supply chain's recycling capabilities. This is for two reasons: First, on the technical side, as Proposition 4 highlighted, whether the supplier's first-best and equilibrium efforts differ is heavily influenced by parameter β. Second, recall that our study firm recently formed a joint venture with the forging supplier to facilitate a higher rate recycling of scrap material, and thus it is of significant managerial importance to assess whether (and if so, to what extent) enhanced recycling improves the profit generating potential of their buyer-supplier relationship. Consequently, our focus in the rest of this subsection remains on the comparative statics of $\Delta(\beta)$ with respect to β (as well as other problem parameters). To this end, one must analyze changes in both $\Pi^{fb}(\beta)$ and $\Pi^*(\beta)$; the next lemma focuses on the former:

Lemma 1. $\Pi^{fb}(\beta)$ *increases with an increase in β.*

Recall from Proposition 1 that first-best effort e^{fb} increases with β as the supplier's input reduction efforts and enhanced recycling in the supply chain are substitutes. Consequently, when β is high the input material costs are lower, and the supply chain does not necessitate as much input reduction as it would otherwise. This, in turn, induces a low e^{fb}, and yields input reduction cost savings for the supply chain whilst increasing salvage revenues in expectation. The net effect for the supply chain would be an increase in first-best profit.

Comparatively, how equilibrium profit $\Pi^*(\beta)$ changes with β is more complex. Even though enhanced recycling induces (weakly) monotone responses in the buyer's compensation rate choice and the ensuing effort on supplier's part (see Proposition 2), the net impact on $\Pi^*(\beta)$ is not always monotone. Consequently, an increase in β may improve or worsen decentralization cost. We characterize the comparative statics of decentralization cost $\Delta(\beta)$ with respect to recycling rate β in the next proposition:

PROPOSITION 5. *Recall $\underline{\beta}$ as in Proposition 4, and define*

$$\overline{\beta} \triangleq \max\Big\{\frac{p_v - k_2}{\Delta_p} - \Big(\frac{(2m_1x_0 + m_2 - r)k_1}{(k_1+m_1)\Delta_p}\Big)^+, 0\Big\}.$$

We have $\overline{\beta} \geq \underline{\beta}$. Also, let three complementary intervals denoted by $\mathbb{B}_i$, $i = \{L, M, H\}$, be defined as

$$\mathbb{B}_L \triangleq \big[0, \underline{\beta}\big] \quad \mathbb{B}_M \triangleq \big[\underline{\beta}, \overline{\beta}\big] \quad \mathbb{B}_H \triangleq \big[\overline{\beta}, 1\big].$$

Then, we have the following statements:

(i) For $\beta \in \mathbb{B}_L$, decentralization cost $\Delta(\beta)$ decreases monotonically with β as $\alpha^* = 0$ and $e^{fb} < e^*$. Furthermore, if $\beta = \underline{\beta}$ then $\Delta(\beta) = 0$.

(ii) For $\beta \in \mathbb{B}_M$, decentralization cost $\Delta(\beta)$ increases or decreases with β as $\alpha^* = 0$ and $e^{fb} > e^*$.

(iii) For $\beta \in \mathbb{B}_H$, decentralization cost $\Delta(\beta)$ decreases monotonically with β as $\alpha^* > 0$ and $e^{fb} > e^*$. Furthermore, $\Delta(1) = \inf \Delta(\beta)$ for $\beta \in \mathbb{B}_H$ if $\Delta(1) > 0$; otherwise, there exists $\breve{\beta} < 1$ satisfying $\Delta(\breve{\beta}) = 0$.

Proposition 5 implies three distinct ranges for β, within each of which $\Delta(\beta)$ behaves differently when β changes. Furthermore, the relative magnitudes of $\underline{\beta}$ and $\overline{\beta}$ yield four distinct cases, which we illustrate in Figures 3 and 4.

Case(i): $(0 < \underline{\beta} < \overline{\beta})$ We highlight this scenario in Figure 3(a), wherein all three intervals $\mathbb{B}_L$, $\mathbb{B}_M$ and $\mathbb{B}_H$ are non-empty and lie within [0,1]. Recall from Proposition 4 that when β is high, first-best effort e^{fb} exceeds equilibrium effort e^*. This is because average input material cost $\bar{p}(\beta)$ is too low, which disincentivizes the supplier's input reduction efforts. As a result, the buyer should offer a positive compensation rate at equilibrium, i.e., $\alpha^* > 0$, which increases further if β increases. In effect, a higher β will reduce the supplier's equilibrium effort even more, yielding savings in input reduction effort costs in addition to lower unit input material costs. The buyer, on the other hand, enjoys additional salvage revenues as processing a larger blank generates more scrap. These savings dominate the additional material removal costs for the buyer, and the costs associated with larger material requirements, which yields an improved equilibrium profit $\Pi^*(\beta)$ for the supply chain. Evidently, if β falls within $\mathbb{B}_H$, the equilibrium profit $\Pi^*(\beta)$ increases even faster with β than the first-best profit $\Pi^{fb}(\beta)$ does, and thus decentralization cost $\Delta(\beta)$ decreases. Furthermore, as $\beta \in \mathbb{B}_H$ increases the supply chain does not need as much input reduction, either, causing the effort differential $e^{fb} - e^*$ to get smaller. (In other words, e^{fb} decreases faster with β than e^* does.) Consequently, if the buyer's material removal cost is small enough, it is even possible that decentralization cost vanishes at a sufficiently high recycling rate, at which point both the buyer and the supplier are content with the status-quo design, i.e., $e^{fb} = e^* = 0$ yielding $x = x_0$.

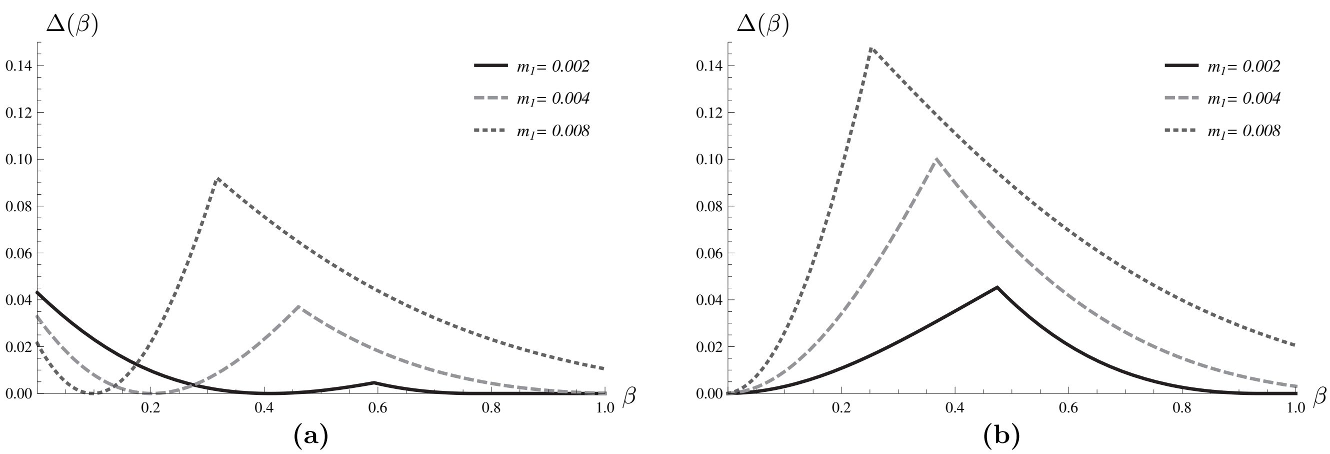

Figure 3 **Decentralization cost $\Delta(\beta)$ vs. recycling rate β with the following parameters:** $V_b + V_s = 200$, $x_o = 101$, $p_v = 2$, $p_s = 0.5$, $r = 0.25$, $k_1 = 0.005$, $k_2 = 1$, **and (a)** $m_2 = 0$, **(b)** $m_2 = 0.2$. **In each panel, we permit** $m_1 \in \{0.002, 0.004, 0.008\}$.

In contrast with the dynamics for the scenario satisfying $\beta \in \mathbb{B}_H$, the buyer does not compensate the supplier's input reduction efforts, i.e., $\alpha^* = 0$, when the recycling rate β is lower (than $\overline{\beta}$) and within interval $\mathbb{B}_L \cup \mathbb{B}_M$. In such scenarios, if β is low enough to yield a high average input material cost, i.e., when $\beta \in \mathbb{B}_L$, then the supplier exerts input reduction efforts despite not being compensated by the buyer at all, yielding $e^* > e^{fb}$. In this case, the lower the recycling rate is, the larger the aforementioned effort differential $e^* - e^{fb}$ gets inflating the supplier's input reduction costs. As a result, the decentralization cost $\Delta(\beta)$ increases. Conversely, an increase in $\beta \in \mathbb{B}_L$

induces equilibrium effort e^* to approach first-best effort e^{fb}, where the supplier's and buyer's actions are coordinated at $\beta = \underline{\beta}$ eliminating the decentralization cost.

When the recycling rate falls within $[\underline{\beta}, \overline{\beta}]$, i.e., $\beta \in \mathbb{B}_M$, the supplier's equilibrium response to the buyer's equilibrium strategy of not compensating any input reduction efforts is to exert effort that falls short of the first-best, i.e., $e^* < e^{fb}$. This is because the supplier is content with the status-quo design's high input material requirement due to the relatively higher recycling rate, and the ensuing relatively lower average input material cost. As this effort differential $e^{fb} - e^*$ gets larger with an increase in β, the decentralization cost increases until $\beta = \overline{\beta}$, at which point the buyer starts compensating the buyer's input reduction efforts, i.e., $\alpha^* > 0$.

Also evident in Figure 3(a) are the comparative statics of $\Delta(\beta)$, critical recycling rate thresholds $\underline{\beta}$ and $\overline{\beta}$, and intervals $\mathbb{B}_i$, $i = \{L, M, H\}$ with respect to the buyer's material removal cost. Specifically, as m_1 (i.e., the quadratic coefficient of the buyer's material removal cost,) increases, we observe that both $\underline{\beta}$ and $\overline{\beta}$ gets smaller, effectively shrinking interval $\mathbb{B}_L$ and expanding interval $\mathbb{B}_H$. As a result, decentralization cost $\Delta(\beta)$ becomes more sensitive to changes in the recycling rate, and coordination may not be possible for $\beta \in \mathbb{B}_H$ even when the recycling rate is very high, as the first-best effort remains positive to avoid high material removal costs. (See $\Delta(1) > 0$ on the curve with $m_1 = 0.008$.) For $\beta \in \mathbb{B}_L$, the supplier does not exert any effort without being compensated unless β is very low, and thus a higher m_1 shifts $\underline{\beta}$ to smaller values.

Case(ii): ($\underline{\beta} = 0$ and $\overline{\beta} > 0$; $\Delta(\beta)$ is increasing in $\mathbb{B}_M$) This scenario in Figure 3(b) results as we transition from **Case (i)** to highlight the impact of increasing m_2 (i.e., the linear coefficient of the buyer's material removal cost,) on the aforementioned comparative statics (of $\Delta(\beta)$, $\underline{\beta}$, $\overline{\beta}$, and intervals $\mathbb{B}_i$, $i = \{L, M, H\}$). We find that higher m_2 yields the same dynamics as higher m_1; furthermore, a decreasing $\underline{\beta}$ may even become negative. Consequently, it becomes impossible to achieve coordination at low recycling rates. This is because the supplier refuses to exert any effort without being compensated, whilst the supply chain needs more input material reduction as the buyer needs to overcome high machining costs. As a result the effort differential $e^{fb} - e^*$ remains positive, yielding $\mathbb{B}_L = \emptyset$. We formalize the aforementioned comparative statics of $\underline{\beta}$ and $\overline{\beta}$ with respect to m_1 and m_2 in the next corollary:

COROLLARY 2. *Both $\underline{\beta}$ (when > 0) and $\overline{\beta}$ (when > 0) decrease as m_1 or m_2 increases.*

Case(iii): ($\underline{\beta} = 0$ and $\overline{\beta} > 0$; $\Delta(\beta)$ is decreasing in $\mathbb{B}_M$) We highlight this scenario in Figure 4(a). In this case, similar to **Case(ii)**, and thus the supplier exerts an input reduction effort without compensation even when the recycling rate is very low, i.e, $\mathbb{B}_L = \emptyset$. What is different in this scenario is that the buyer's input reduction cost is significantly higher relative to **Case(i)** and **Case(ii)**, and thus the supplier has to exert a significant input reduction effort at equilibrium to overcome high input material costs when the recycling rate is very low. At first-best, the resulting increase in effort costs are partially negated by the buyer's material removal cost savings, because a higher effort yields a blank with less material removal requirements. However, at equilibrium, the supplier cannot benefit from the buyer's material removal cost savings, yielding a significant effort differential $e^{fb} - e^*$ and thus a high decentralization cost. An increase in the recycling rate, on the other hand, reduces input material costs and necessitates less effort on the buyer's part, yielding input reduction effort cost savings for the supply chain for both the first-best and the equilibrium solutions. As effort differential $e^{fb} - e^*$ shrinks, the equilibrium supply chain profit increases faster than the first-best supply chain profit does, and thus $\Delta(\beta)$ is decreasing in β even when $\beta \in \mathbb{B}_M$.

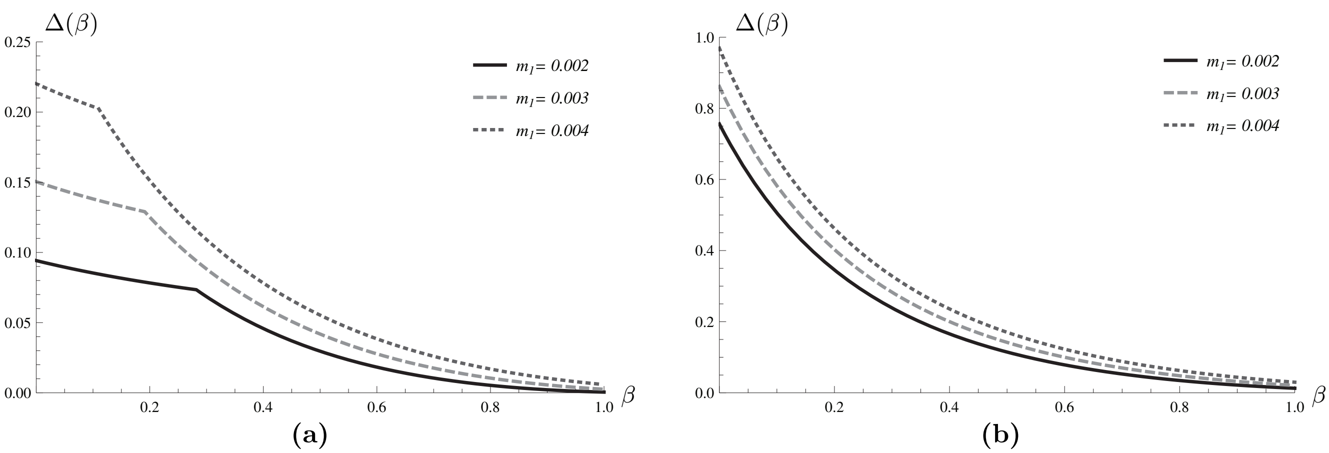

Figure 4 **Decentralization cost $\Delta(\beta)$ vs. recycling rate β with the following parameters:** $V_b + V_s = 250$, $x_o = 101$, $p_v = 2$, $p_s = 0.5$, $r = 0.25$, $k_1 = 0.015$, $k_2 = 1$, **and (a)** $m_2 = 0.5$, **(b)** $m_2 = 1$. **In each panel, we permit** $m_1 \in \{0.002, 0.003, 0.004\}$.

Case(iv): ($\underline{\beta} = \overline{\beta} = 0$) We highlight this scenario in Figure 4(b), which implies $\mathbb{B}_H \equiv [0,1]$. This scenario transpires when $p_v < k_2 + \left(\frac{(2m_1x_0+m_2-r)k_1}{(k_1+m_1)}, 0\right)^+$, a condition that holds when the expression for $\overline{\beta}$ in Proposition 5 returns 0. In other words, as the supplier's input reduction cost parameters (k_1, k_2) and/or the buyer's material removal cost parameters (m_1, m_2) increase, $\overline{\beta}$ gradually declines to 0 causing interval $\mathbb{B}_M$ to collapse to become the empty set. In Figure 4, we highlight the impact the latter of the aforementioned cost effects as m_2 increases to yield the transition from panel (a) to panel (b). (Recall that Corollary 2 also predicts this transition.) In this scenario, a higher material removal cost induces the buyer to always try to incentivize the supplier even when the recycling rate is very low.

5. Analysis: The Asymmetric Information Case

Our analysis in this section considers asymmetric information regarding the supplier's cost to exert a given level effort, i.e., the aforementioned cost is unknown to the buyer, and is the supplier's private information. This scenario applies to settings wherein the buyer and the supplier are either completely unrelated, or the buyer does not have oversight over the supplier's operations and relevant costs. In our study firm's setting, for example, this scenario may involve the buyer contacting with a new supplier instead of the joint venture with the virgin specialty material provider.

We will permit two types of suppliers to facilitate our analysis; a supplier is of type-z with probability γ_z. Therefore, a supplier is either efficient (i.e., type-$\underline{z}$) with probability $\gamma_{\underline{z}} = \gamma$, or inefficient (i.e., type-$\overline{z}$ where $\underline{z} < \overline{z}$) with probability $\gamma_{\overline{z}} = 1 - \gamma$. We use function $K_z(e) = zK(e)$ to capture variations in type-z supplier's input reduction cost, and without loss of generality, we scale $\overline{z}$ to 1. We assume that the buyer knows the distribution of possible supplier types (i.e., probability γ), and the supplier's effort-to-output (i.e., e-to-$\mathbb{E}[X]$) mapping in expectation; however, only the supplier knows its own type and cost $K_z(e_z)$ to exert effort e_z. Similar to our analysis for the symmetric information case, we will first establish a benchmark solution, i.e., the first-best, by considering a setting wherein the buyer and the supplier act together. Then, we will analyze the decentralized equilibrium wherein each party submits decisions independently and sequentially, following the decision timeline we established in Section 4. In both the centralized and decentralized scenarios, nature first determines the supplier's type, and, as before, both the buyer and the supplier are risk-neutral utility maximizers.

5.1. The First-Best Solution

In this subsection, we analyze a centralized decision maker's optimization problem prior to nature resolving the uncertainty regarding the supplier's type, with the objective of maximizing the sum of the buyer's and the supplier's profits. As there are two possible types for the supplier, we revise our notation accordingly; we denote by e_z and $\Pi_s^z(x, e_z)$ the type-z supplier's effort and profit, respectively, and by $\Pi_b^z(X, e_z)$ the buyer's profit. (Note that functions $\Pi_s^z(x, e_z)$ and $\Pi_b^z(X, e_z)$ only differ from their counterparts in the previous section for including a supplier type specific effort cost $K_z(e_z)$.) As all transfers remain within the supply chain, characterizing the first-best solution requires finding the first-best effort vector $\mathbf{e}^{fb} = (e_{\underline{z}}^{fb}, e_1^{fb})$, which satisfies

$$\mathbf{e}^{fb} \triangleq \arg\max_{\mathbf{e} \geq \mathbf{0}} \mathbb{E}_z \Big[\mathbb{E}_X \big[\Pi_s^z(X, e_z) + \Pi_b^z(X, e_z) \big] \Big].$$

One can verify easily that the supply chain's objective is jointly concave in $e_{\underline{z}}$ and e_1, and is also separable. Consequently, the first-best is characterized by the Karush-Kuhn-Tucker (KKT) conditions, which we formally state next:

Proposition 6. *The first best effort vector* $\mathbf{e}^{fb}$ *satisfies*

$$e_z^{fb} = \max\left\{\frac{2m_1x_0 + m_2 - zk_2 + \bar{p}(\beta) - r}{2(m_1 + zk_1)}, 0\right\}, \forall z \in \{\underline{z}, 1\} \tag{10}$$

for any given β*, and is non-increasing in* β *for all* $z \in \{\underline{z}, 1\}$*.*

Proposition 6 reflects that information asymmetry is inconsequential in this centralized scenario; in other words, once the nature determines the supplier's type, that information is known to both the buyer and the supplier. Thus, the supply chain demands that an efficient supplier exerts effort $e_{\underline{z}}^{fb}$, and an inefficient supplier exerts effort e_1^{fb}. Effort e_1^{fb} coincides with the first-best effort as per equation (5) for the symmetric case. Proposition 6 further highlights that the first-best effort for each supplier type evolves with changes in problem parameters (e.g., x_0, m_1, m_2, k_1, k_2, $\bar{p}(\beta)$) the same way the first-best effort for the symmetric case changes; most notably that the recycling rate and the input reduction efforts are substitutes. One notable difference is that e_z^{fb} decreases with z, i.e., an efficient supplier exerts more effort at the first-best than an inefficient supplier does. This is because the same amount of input reduction spending can finance a higher exerted effort for the efficient type, which yields a smaller forging in expectation and results in more cost savings for the supply chain. These savings materialize in the form of less input material costs for the supplier and less material removal costs for the buyer.

5.2. The Decentralized Supply Chain

In this subsection, we analyze the scenario wherein the buyer and the supplier act individually, and the supplier has private information regarding what cost it incurs to exert a given effort level. The sequence of events remains the same as was in the symmetric information case, i.e., the buyer offers contract terms, and then the supplier responds by exerting effort. What is different in this asymmetric information case is that, before offering contract terms, the buyer only knows that the supplier's effort cost is $K_z(e_z)$ with probability γ_z for all $z \in \{\underline{z}, 1\}$. Thus, the buyer must consider a *menu* of contracts of the form

$$t_z(X, \omega_z, \alpha_z) = \omega_z + \alpha_z(x_o - X) \tag{11}$$

to a supplier if that supplier declares as a type-z supplier. Evidently, it is possible that a type-z supplier may choose to act in a manner that contrasts the buyer's expectation. The buyer may prevent such behavior by designing a contract that is *truth-inducing*, i.e., it would be optimal for a type-z supplier to act as a type-z supplier would. In the rest of this paper, our equilibrium analysis will focus on such truth-inducing contracts. As before, we employ backward induction to characterize the players' equilibrium decisions; ω_z^* and α_z^* for the buyer, and supplier's best-response effort

$e_z^{br}(\omega_z, \alpha_z, \beta)$, which reveals its type truthfully. For brevity, we denote type-z supplier's optimal effort by e_z^{br}, and first solve its problem of finding

$$e_z^{br} \triangleq \arg\max_{e_z \geq 0} \mathbb{E}_X\left[\Pi_s^z(X, e_z)\right],$$

for which the next proposition characterizes a solution.

PROPOSITION 7. *A type-z supplier's optimal effort e_z^{br} satisfies*

$$e_z^{br} = \max\left\{\frac{\alpha_z - zk_2 + \bar{p}(\beta)}{2zk_1}, 0\right\} \tag{12}$$

for any given α_z and β, and does not depend on ω_z. Furthermore, e_z^{br} is non-decreasing in α_z, non-increasing in β and z.

Proposition 7 highlights that the buyer can induce different levels of effort on suppliers' part by discriminating by the compensation rate α_z. However, e_z^{br} also depends on and is non-increasing in z, and thus it is not a guarantee that a higher compensation rate implies higher effort. More specifically, if the buyer offered the same compensation rate to each supplier type, i.e., $\alpha_{\underline{z}} = \alpha_1$, an inefficient supplier would exert less effort, which the buyer may counter by offering a higher compensation rate to that inefficient supplier, i.e., $\alpha_1 > \alpha_{\underline{z}}$. But then, an efficient supplier would be tempted to declare as the inefficient type, which the buyer's optimization should take into account. Next, we formulate the buyer's optimization subject to *incentive compatibility* constraints to ensure truth-telling by a supplier regarding its type:

$$\begin{aligned} \max_{(\omega_z, \alpha_z) \geq 0, \forall z \in \{\underline{z}, 1\}} \quad & \mathbb{E}_z\left[\mathbb{E}_X\left[\Pi_b^z(X, e_z^{br})\right]\right] \\ \text{subject to } & \mathbb{E}_X\left[\Pi_s^z(X, e_z^{br})\right] \geq \max_{z \in \{\underline{z}, 1\}} \mathbb{E}_X\left[\Pi_s^z(X, e_z^{br})\right] \quad, \forall z \in \{\underline{z}, 1\} \end{aligned} \tag{13}$$

Note that the buyer's optimization program (13) permits two classes of equilibria based on the buyer's contract choice: *Pooling equilibria* where the buyer sets $\omega_{\underline{z}} = \omega_1$ and $\alpha_{\underline{z}} = \alpha_1$, and *separating equilibria* where the buyer sets $\omega_{\underline{z}} \neq \omega_1$ and/or $\alpha_{\underline{z}} \neq \alpha_1$. Even though it is not possible to obtain a full characterization of each of the aforementioned equilibria for the original setting we consider, we present notable structural properties in the next proposition.

PROPOSITION 8. *Denoting by $(\omega_{\underline{z}}^*, \alpha_{\underline{z}}^*)$ and (ω_1^*, α_1^*) the buyer's optimal contract parameters, the following statements hold true:*

(i) Each separating equilibrium supports $\alpha_{\underline{z}}^* \leq \alpha_1^*$ and $\omega_{\underline{z}}^* \geq \omega_1^* = 0$.

(ii) Each pooling equilibrium supports $\alpha_{\underline{z}}^* = \alpha_1^*$ and $\omega_{\underline{z}}^* = \omega_1^* = 0$.

(iii) Any equilibrium supports $e_{\underline{z}}^{br} > e_1^{br}$ (when both are positive).

(iv) When the buyer's material removal cost is linear, i.e., $m_1 = 0$, only a pooling equilibrium sustains optimally.

Proposition 8(i) highlights that if a separating equilibrium sustains optimally, the buyer would not make an upfront payment to the inefficient supplier (as the best-response effort does not depend on ω_z) and only subsidize its forging-size-reduction effort. This compensation rate would be even higher than the compensation rate the efficient supplier would be offered, so that the inefficient supplier can overcome its cost disadvantage. On the other hand, the buyer would still offer a fixed payment to efficient supplier, even though such a payment does not induce more effort on this supplier's part. Instead, the upfront payment ensures that the efficient supplier would not be tempted to declare as the inefficient type to receive higher compensation rate.

Proposition 8(ii) highlights the scenario wherein a pooling equilibrium sustains optimally. In this case, as noted in Proposition 7, each supplier types's best-response effort does not depend on the upfront payment. Also recall that the efficient type exerts more effort when the compensation rates are the same, as it would be according to a contract that induces a pooling equilibrium where $\alpha^*_{\underline{z}} = \alpha^*_1$. Thus, the buyer hopes the supplier is the efficient type, but is not compelled to pay information rent to induce truth telling. As a result, the buyer optimally sets the upfront payment to 0, and the optimal compensation rate at a level which would induce the optimal best-response effort from an average supplier with forging reduction cost $\mathbb{E}_z\big[K_z(e_z)\big]$.

For cases where a pooling equilibrium sustains Proposition 8(iii) is a corollary to Proposition 7. On the other hand, when a separating equilibrium sustains, Proposition 8(iii) shows that despite a higher compensation rate the inefficient type does not exert as much effort as the efficient type would. This is because, if the buyer compensates the inefficient type at a rate that would ensure $e^{br}_{\underline{z}} = e^{br}_1$, then it would not have enough funds to pay the efficient type's information rent. As a result, at both equilibriums, the efficient type delivers a more near-net-shape forging in expectation. Finally, Proposition 8(iv) highlights a special scenario with linear material removal costs, wherein only a pooling equilibrium can sustain.

5.3. Coordination Issues, and the Impact of Recycling

Our analysis of supply chain efficiency when there is no information asymmetry between the supplier and the buyer showed that enhanced recycling affects decentralization cost in a non-monotone way. Nevertheless, under certain conditions, a higher recycling rate always reduces decentralization cost; for example, when the supplier's input reduction cost and the buyer's material removal cost are both high. In this section, we investigate how information asymmetry influences these aforementioned dynamics, and whether it is still possible that the supply chain performance can be improved by enhanced recycling. Specifically, we will show that enhanced recycling can reduce decentralization cost only in some problem scenarios. We first characterize supply chain efficiency dynamics for one such scenario, the case with linear material removal cost, which we highlighted in Proposition 8(iv).

PROPOSITION 9. *When the buyer's material removal cost is linear, i.e.,* $m_1 = 0$*, the following statements hold true:*

(i) If $e_{\underline{z}}^{fb}(\beta) = 0$*, then* $\Delta(\beta) = 0$*.*

(ii) If $e_{\underline{z}}^{fb}(\beta) > 0$*, then* $\Delta(\beta) > 0$*. Nevertheless, when* $e_1^{fb}(\beta) = 0$*,* $\Delta(\beta)$ *decreases as* β *increases.*

Proposition 9 highlights that enhanced recycling can improve supply chain performance even when a pooling equilibrium sustains despite the supplier's private cost information. Furthermore, comparative statics of $\Delta(\beta)$ with respect to β is influenced by each supplier type's first-best effort. To interpret Proposition 9(i), recall from Proposition 6 that the efficient supplier's first-best effort strictly (weakly) exceeds the inefficient supplier's first-best effort, i.e., $e_{\underline{z}}^{fb}(\beta) \geq e_1^{fb}(\beta)$. Therefore, $e_{\underline{z}}^{fb}(\beta) = 0$ implies $e_1^{fb}(\beta) = 0$, in which case decentralization is inconsequential, and supply chain coordination results.

On the other hand, in the scenario prescribed by Proposition 9(ii) when the efficient supplier's first-best effort is positive, i.e., $e_{\underline{z}}^{fb}(\beta) > 0$, Proposition 6 dictates that the inefficient supplier's first-best effort could be positive or zero. In the former subcase with $e_1^{fb}(\beta) > 0$, both supplier types' equilibrium efforts differ from (and are less than) their first-best counterparts, and thus the supply chain remains inefficient, i.e., $\Delta(\beta) > 0$. This is a scenario wherein the involved parties play a pooling equilibrium with a null contract, i.e., $t = 0$, and each supplier exerts a positive effort despite receiving no payment in order to compensate high input material costs due to a low recycling rate. Nevertheless, such a positive efforts remain below the requisite first-best levels, and thus decentralization cost ensues. More recycling further distorts supply chain efficiency, i.e., $\Delta(\beta)$ increases as β increases, because a higher β discourages input reduction efforts on each supplier type's part, thus increasing effort-differential $e_z^{fb}(\beta) - e_z^*(\beta)$ for each $z \in \{\underline{z}, 1\}$. In contrast with the subcase with $e_1^{fb}(\beta) > 0$ (and $e_{\underline{z}}^{fb}(\beta) > 0$), enhanced recycling reduces decentralization cost when $e_1^{fb}(\beta) = 0$. This is because, in this subcase, the buyer can adjust the pooling contract terms to influence *only* the efficient supplier's equilibrium effort level $e_{\underline{z}}^*(\beta)$, which remains positive, without distorting the inefficient supplier's equilibrium effort level $e_1^*(\beta)$, which matches its first-best value of zero.

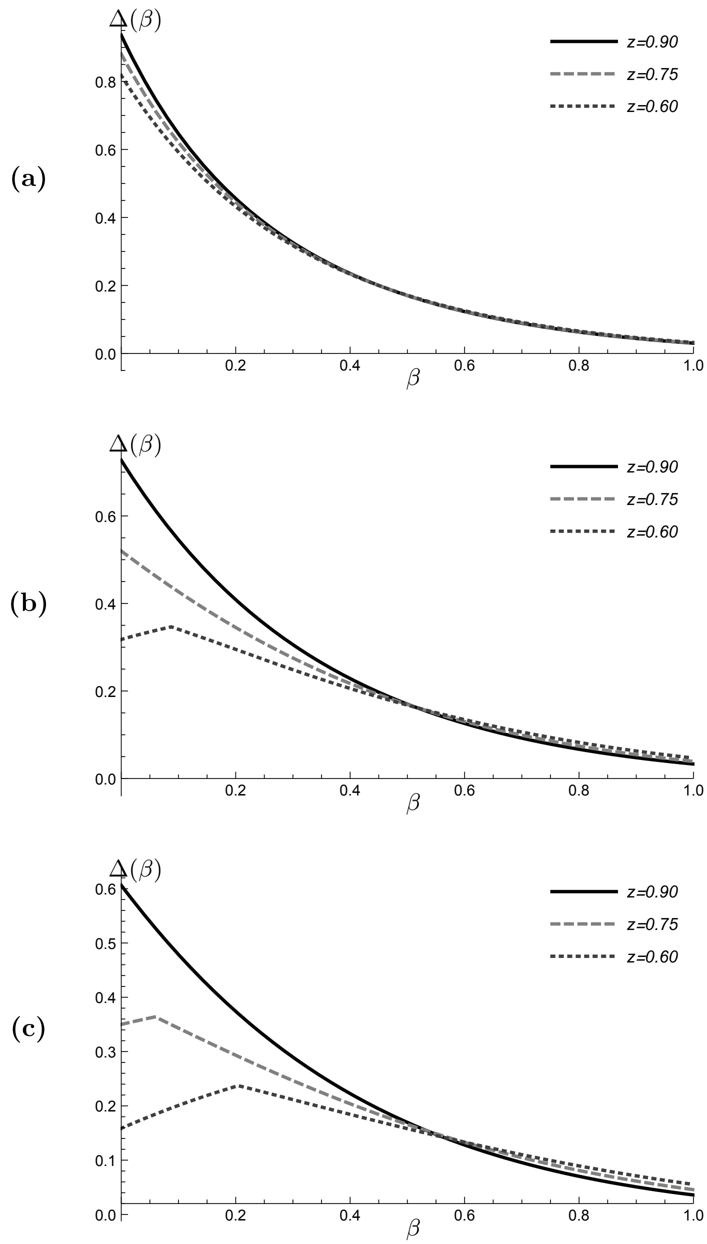

Figure 5 **Decentralization cost** $\Delta(\beta)$ **vs. recycling rate** β **with the following parameters:** $V_b + V_s = 250$, $x_o = 101$, $p_v = 2$, $p_s = 0.5$, $r = 0.25$, $k_1 = 0.015$, $k_2 = 1$, $m_1 = 0.002$, $m_2 = 1$, **and (a)** $\gamma = 0.05$, **(b)** $\gamma = 0.50$, **(c)** $\gamma = 0.95$. **In each panel, we permit** $\underline{z} \in \{0.60, 0.75, 0.90\}$.

In Figure 5, we illustrate the aforementioned dynamics Proposition 9 prescribes for scenarios when the buyer's material removal cost is not necessarily linear (and thus, a separating equilibrium is also possible.) The instances we illustrate differ from each other in the likelihood that the buyer would contract with an efficient supplier. Specifically probability γ increases gradually from 5% to 95%. In each panel, we consider three levels of unit input-reduction cost advantage for the efficient type ranging from 10% to 40%, i.e., we permit $\underline{z} \in \{0.60, 0.75, 0.90\}$.

We kept all other instance parameters the same as they were in Figure 4(b) to ensure a meaningful comparison between the symmetric information and the asymmetric information cases. Therefore, the decentralization cost dynamics we observe in Figures 4(b) and 5(a) are almost identical. In the former, there is no information asymmetry and the supplier is inefficient, i.e., $\gamma = 1$ and $\underline{z} = 1$, whereas in the latter, there is only a low probability ($\gamma = 0.05$) of contracting with an efficient supplier, i.e., $\underline{z} < 1$. We further observe that, in this scenario with high input-reduction cost for the supplier and high material removal cost the buyer, the null contract never sustains in equilibrium, and enhanced recycling improves decentralization cost.

We also observe in Figure 5(a) that, when it is likely to contract with an inefficient supplier, the decentralization cost is not sensitive to how cost-efficient the other supplier is. However, as it becomes more likely to contract with the efficient supplier, (i.e., as γ increases), we observe striking differences as illustrated in panels (b) and (c). Firstly, as anticipated, $\Delta(\beta)$ decreases with an increase in γ (for each β). Secondly, the efficient supplier's cost advantage parameterized by $\underline{z} = 1$ becomes more consequential. If the supply chain cannot recycle effectively, i.e., low β, then $\Delta(\beta)$ decreases as the efficient supplier's cost advantage increases. On the other hand, if β is high, $\Delta(\beta)$ increases (yet, remains comparable) as $\underline{z}$ decreases. Thirdly, and most strikingly, as it becomes more likely for the buyer to contract with an efficient supplier, information asymmetry becomes significantly consequential and enhanced recycling can induce an increase in supply chain inefficiency when β is sufficiently low, which we illustrate this in panels (b) and (c) of Figure 5. This is because, in anticipation of the contracting with an efficient supplier which is motivated to engage in input-reduction (due to the low recycling rate and the efficient supplier's input-reduction cost advantage), the buyer offers the null contract at equilibrium, thus discouraging supplier type's input reduction efforts. As a result, decentralization cost $\Delta(\beta)$ increases as the supply chain recycles more effectively up until β reaches a threshold at which the null contract is no longer the equilibrium. Only then, further recycling can improve decentralization cost.

6. Discussion, Managerial Insights and Conclusion

For manufacturing industries that involve specialty-material-component production, input material spending is the main (variable) cost driver due to high spot prices. Airline industry is one prominent example, where major players rely heavily on titanium alloys as they try to manufacture light-weight components. Our interactions with one such manufacturing firm provided us with the opportunity to assess what innovations these firms seek to lower their input material costs. The status-quo process for this study firm involves procuring a standard size (large) forging from the alloy-producer, and removing excess material from it via various machining operations until it is reduced to a geometric form consistent with the design specifications of a particular airplane component. The weight of the procured forgings relative to the final components ("buy-to-fly ratio") is tremendously high, which implies generating a significant amount of scrap material during the machining process. To mitigate the resulting high input material costs, the study firm formed a joint venture with its supplier, the largest titanium mill in the world, to efficiently recycle scrap material to forging-production. Alternatively, the design engineers at the study firm suggest the supplier may provide forgings that are more "near-net-shape" (i.e., with lower buy-to-fly ratio); however, this approach requires a significant investment with uncertain yield on supplier's part. Therefore, the latter alternative creates an agency problem, which may imply information asymmetry issues. In this paper, we assessed the value of the two aforementioned innovative approaches our study firm had taken under consideration, i.e., enhanced recycling to reduce average input material cost vs. incentivizing the supplier to encourage input material reduction.

In our analysis, we considered the relationship between a supplier of specialty material forgings and a buyer that manufactures final components after extensive material removal as per design specifications. We permitted the buyer and the supplier to contract on the effort the supplier can induce to achieve forging size reduction, and studied how the resulting equilibrium changes as the ensuing supply chain increases its recycling capabilities. We measured supply chain performance by assessing decentralization cost relative to first-best outcomes in both symmetric and asymmetric information scenarios. In the former scenario, the only uncertainty is regarding how near-net-shape the resulting forging is, whereas, in the latter case, we assumed the supplier's aforementioned forging size reduction cost is unknown to the buyer.

We found that the supplier's input reduction efforts and enhanced recycling across the supply chain interact in non-intuitive ways, thus inflicting a non-monotone effect on decentralization cost. In the symmetric information scenario, when the recycling rate is low, the supplier exerts more effort than the first-best dictates even though the buyer does not compensate forging-size reduction efforts by offering a null contract. In this case, enhanced recycling improves supply chain performance. On the other hand, at higher recycling rates, enhanced recycling disincentivizes the supplier, and thus the supply chain performance worsens, unless the buyer compensates the supplier's forging size reduction efforts. As the recycling rate increases further, the supply chain can be coordinated provided the buyer's material removal cost and the supplier's effort cost are both reasonably low, and the supplier is properly incentivized. Consequently, the managerial implication for our study firm (corresponding to the buyer in our model) would be to participate in a supply chain with either *superb* or *insufficient* recycling capabilities. The former strategy works if the buyer and the supplier are closely related and are willing to fairly split the economic value their relationship can create. If the involved parties are disjoint decision makers, then the latter recycling scenario is more beneficial to the buyer. In this case, the supply chain is worse-off, yet the supplier is hurt disproportionately as it is already motivated to engage in forging-size-reduction to improve its input material costs, while the buyer offers no compensation for such efforts despite its improved material removal costs.

We observe changes in insights when agency issues arise and the buyer does not know the supplier's input reduction cost. Specifically, we find that a positive decentralization cost surely results when there is not enough recycling in the supply chain. This inefficiency is particularly high when the supplier types are not too distinct from one another, and the chances of contracting with an efficient type is low. In such scenarios, utilizing more recycled input always improves supply chain performance, justifying investments geared towards enhanced recycling such as the one undertaken by our study firm. On the other hand, when there is a high probability of contracting with a distinctly more efficient supplier, we find the impacts of improved recycling to be two-fold. If the recycling level is above a threshold, we find that the buyer should subsidize supplier's forging size reduction efforts, which helps improve supply chain performance. In contrast, when the recycling level is below the same threshold, the buyer bets on the possibility of contracting with an efficient supplier, (which, by itself, would be motivated to engage in input reduction) and withholds any compensation. In such cases, supply chain inefficiency remains at a high level unless the recycling level is sufficiently low to motivate each supplier type to invest in input reduction willingly due to high input material costs.

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APPENDICIES

A. Proofs in Section 4

A.1. Proof of Proposition 1

We define $\pi_{sc}(e) \equiv \mathbb{E}[\Pi_b(X,e)+\Pi_s(X,e)] = V_b + V_s + r[x_0 - e - 1] - M(x_0 - e) - K(e) - \bar{p}(\beta)[x_0 - e] - \beta[p_v - p_s]$, and take its 1st and 2nd order derivatives to obtain

$$\pi'_{sc}(e) = 2m_1[x_0 - e] + m_2 - r - 2k_1 e - k_2 + \bar{p}(\beta), \text{ and } \pi''_{sc}(e) = -2[m_1 + k_1].$$

As $\pi''_{sc}(e) < 0$, function $\pi_{sc}(e)$ must be concave. Furthermore, as $e = x_0 - 1$ cannot be optimal by assumption, we have $\pi'_{sc}(x_0 - 1) < 0$. Then, the optimal effort for the central planner e^{fb} is either the solution for $\pi'_{sc}(e) = 0$ or 0.

To prove the monotonicity of $e^{fb}(\beta)$ in β, we calculate $\frac{de^{fb}(\beta)}{d\beta}$ when $e^{fb}(\beta) > 0$ to obtain

$$\frac{de^{fb}(\beta)}{d\beta} = \frac{\bar{p}'(\beta)}{2(m_1 + k_1)} < 0,$$

which follows from the buyer's and the supplier's costs being convex, and $\bar{p}'(\beta) = p_s - p_v < 0$.

A.2. Proof of Proposition 2

We define $\pi_s(e) \equiv \mathbb{E}[\Pi_s(X,e) + t(X,w,\alpha)] = V_s + w + \alpha a - K(e) - \bar{p}(\beta)[x_0 - e] - \beta[p_v - p_s]$, and take its 1st and 2nd order derivatives to obtain

$$\pi'_s(e) = \alpha - 2k_1 e - k_2 + \bar{p}(\beta), \text{ and } \pi''_s(e) = -2k_2.$$

As $\pi''_s(e) < 0$, function $\pi_s(e)$ must be concave. Furthermore, as $e = x_0 - 1$ cannot be optimal by assumption, we have $\pi'_s(x_0 - 1) < 0$. Then, the supplier's best response effort e^{br} is either the solution for $\pi'_s(e) = 0$ or 0.

To prove the monotonicity of $e^{br}(\beta,\alpha)$ in β and α, we calculate 1st order partial derivatives of $e^{br}(\beta,\alpha)$ to obtain

$$\frac{\partial e^{br}(\alpha,\beta)}{\partial \beta} = \frac{\bar{p}'(\beta)}{2k_1} < 0 \text{ and } \frac{\partial e^{br}(\alpha,\beta)}{\partial \alpha} = \frac{1}{2k_1} > 0,$$

which follow from the buyer's and the supplier's costs being convex, and $\bar{p}'(\beta) = p_s - p_v < 0$.

A.3. Proof of Corollary 1

Note in Proposition 2 that the supplier's best response effort e^{br} does not depend on fixed payment ω, implying ω must be non-positive. Furthermore, we assumed the supplier has limited liability, i.e., $t \geq 0$, which yields $\omega^* = 0$.

A.4. Proof of Proposition 3

We define $\pi_b(\alpha) \equiv \mathbb{E}[\Pi_b(X, e^{br}(\alpha,\beta)) - t(X,0,\alpha)] = V_b + r[x_0 - e^{br} - 1] - M(x_0 - e^{br}) - \alpha e^{br}$, where $e^{br} = e^{br}(\alpha,\beta)$. As $e^{br}(\alpha,\beta) = 0$ for $\alpha < k_2 - \bar{p}(\beta)$, the buyer's profit must be constant for all parameters satisfying $\alpha < k_2 - \bar{p}(\beta)$. Therefore, it is sufficient to focus on $\alpha \geq \bar{p}(\beta) + k_2$, for which $e^{br}(\alpha,\beta) > 0$.

To show that $\pi_b(\alpha)$ is unimodal in α, we calculate the 1st order derivative of $\pi_b(\alpha)$ with respect to α to obtain

$$\pi'(\alpha) = \frac{de^{br}}{d\alpha}\left[2m_1(x_0 - e^{br}) + m_2 - r - \alpha\right] - e^{br} = \frac{1}{2k_1}\left[2m_1(x_0 - e^{br}) + m_2 - r - 2\alpha - e^{br}k_1\right].$$

We further define $\ell(\alpha) = 2m_1(x_0 - e^{br}) + m_2 - r - \alpha - 2e^{br}k_1$. As $\pi_b'(\alpha) > 0$ if and only if $\ell(\alpha) > 0$, function $\pi_b(\alpha)$ must be unimodal in α if $\ell(\alpha)$ is decreasing in α. Indeed, we have

$$\ell'(\alpha) = -(2 + m_1/k_1) < 0.$$

Unimodularity of $\pi_b(\alpha)$ implies that the optimal compensation rate must satisfy $\alpha^* = \max\left\{\alpha_1(\beta), \alpha_2(\beta)\right\}$. Specifically, we have either $\alpha^* = \alpha_1(\beta)$ where $\alpha_1(\beta)$ satisfies $\ell(\alpha_1(\beta)) = 0$ when $\ell(\max\{0, k_2 - \bar{p}(\beta)\}) > 0$, or $\alpha^* = \alpha_2(\beta) = \max\{0, k_2 - \bar{p}(\beta)\}$ when $\ell(\max\{0, k_2 - \bar{p}(\beta)\}) \leq 0$. To find $\alpha_1(\beta)$ in closed form, we solve $\ell(\alpha) = 0$ yielding

$$\alpha_1(\beta) = k_2 - \bar{p}(\beta) + \frac{k_1}{2k_1 + m_1}\Big(2m_1x_0 + m_2 - r + \bar{p}(\beta) - k_2\Big). \tag{14}$$

Finally, $e^* = e^{br}(\alpha^*)$, and the monotonicity of α^* and e^* follow from $\bar{p}'(\beta) = p_s - p_v < 0$.

A.5. Proof of Proposition 4

We first consider the case where $\alpha^* > 0$. In this case, we have $e^{fb} = (1 + \frac{k_1}{k_1+m_1})e^*$, and thus $e^{fb} \geq e^*$.

When $\alpha^* = 0$, we have $e^* = \frac{\bar{p}(\beta)-k_2}{2k_1}$. As $e^* = 0$ for $\beta > \frac{p_v-k_2}{\Delta_p}$, we must have $e^{fb} \geq e^*$ for $\beta > \frac{p_v-k2}{\Delta_p}$. For $\beta \leq \frac{p_v-k_2}{\Delta_p}$, we define

$$h(\beta) \equiv e^{fb} - e^* = \max\left\{\frac{2m_1x_0 + m_2 - r}{2(k_1 + m_1)} - \frac{m_1\left[\bar{p}(\beta) - k_2\right]}{2k_1(k_1 + m_1)}, -\frac{\bar{p}(\beta) - k_2}{2k_1}\right\},$$

which increases with β as $\bar{p}'(\beta) = p_s - p_v < 0$.

If $2m_1x_0 + m_2 - r < 0$, then we have $h(\beta) < 0$ for all $\beta \leq \frac{p_v-k2}{\Delta_p}$, which implies that $\underline{\beta} = \frac{p_v-k2}{\Delta_p}$. If, on the other hand, $2m_1x_0 + m_2 - r > 0$ holds, then there must exist a critical recycling rate $\beta^o < \frac{p_v-k2}{\Delta_p}$ which solves $h(\beta) = 0$. Therefore, $\underline{\beta}$ satisfies $\underline{\beta} = \max\{\beta^o, 0\}$, where

$$\beta^o = \frac{p_v - k_2}{\Delta_p} - \frac{(2m_1x_0 + m_2 - r)k_1}{(k_1 + m_1)\Delta_p}.$$

A.6. Proof of Lemma 1

Rewriting the supply chain profit at first-best as

$$\Pi^{fb}(\beta) = V_b + V_s + r[x_0 - e^{fb} - 1] - M(x_0 - e^{fb}) - K(e^{fb}) - \bar{p}(\beta)[x_0 - e^{fb}] - \beta[p_v - p_s],$$

we calculate its 1st order derivative with respect to β to obtain

$$\frac{d\Pi^{fb}(\beta)}{d\beta} = \frac{de^{fb}}{d\beta}\left[2m_1[x_0 - e^{fb}] + m_2 - r - 2k_1e^{fb} - k_2 + \bar{p}(\beta)\right] - \bar{p}'(\beta)[x_0 - 1 - e^{fb}].$$

Then, using equation (5), we obtain $\frac{d\Pi^{fb}(\beta)}{d\beta} = \bar{p}'(\beta)[x_0 - 1 - e^{fb}] \geq 0$, where the (weak) inequality follows from $e^{fb} \leq x_0 - 1$ and $\bar{p}'(\beta) = p_s - p_v < 0$.

A.7. Proof of Proposition 5

In order to show the monotonicity of function $\Delta(\beta)$, we use

$$\Pi^{fb}(\beta) = V_b + V_s + r[x_0 - e^{fb} - 1] - M(x_0 - e^{fb}) - K(e^{fb}) - \bar{p}(\beta)[x_0 - e^{fb}] - \beta[p_v - p_s], \text{ and}$$
$$\Pi^{*}(\beta) = V_b + V_s + r[x_0 - e^{br} - 1] - M(x_0 - e^{br}) - K(e^{br}) - \bar{p}(\beta)[x_0 - e^{br}] - \beta[p_v - p_s].$$

Then, we calculate the 1st order derivative of $\Delta(\beta)$ to obtain

$$\frac{d\Delta(\beta)}{d\beta} = -\frac{\frac{d\Pi^*(\beta)}{d\beta}\Pi^{fb}(\beta) - \frac{d\Pi^{fb}(\beta)}{d\beta}\Pi^*(\beta)}{\Pi^{fb}(\beta)^2}$$

Next, by replacing $\Pi^*(\beta)$ with $\Pi^{fb}(\beta)$, and using Lemma 1, we obtain

$$\frac{d\Delta(\beta)}{d\beta} \leq -\frac{\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta}}{\Pi^{fb}(\beta)}.$$

Therefore, it suffices to show that $\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta} \geq 0$ to reach conclusions regarding the monotonicity of function $\Delta(\beta)$. Taking the 1st order derivatives of $\Pi^{fb}(\beta)$ and $\Pi^*(\beta)$, we obtain

$$\begin{aligned}\frac{d\Pi^{fb}(\beta)}{d\beta} &= \frac{de^{fb}}{d\beta}\left[2m_1[x_0 - e^{fb}] + m_2 - r - 2k_1e^{fb} - k_2 + \bar{p}(\beta)\right] - \bar{p}'(\beta)[x_0 - 1 - e^{fb}] \\ &= -\bar{p}'(\beta)[x_0 - 1 - e^{fb}],\end{aligned}$$

as $2m_1[x_0 - e^{fb}] + m_2 - r - 2k_1e^{fb} - k_2 + \bar{p}(\beta) = 0$ at $e = e^{fb}$, and

$$\frac{d\Pi^*(\beta)}{d\beta} = \frac{de^{br}}{d\beta}\left[2m_1[x_0 - e^{br}] + m_2 - r - 2k_1e^{br} - k_2 + \bar{p}(\beta)\right] - \bar{p}'(\beta)[x_0 - 1 - e^{br}].$$

To analyze each part of this proposition, we first establish a relationship between $\underline{\beta}$ and $\overline{\beta}$. Note that $\underline{\beta} = \overline{\beta}$ if $2m_1x_0 + m_2 - r < 0$; otherwise, we have $\underline{\beta} \leq \overline{\beta}$ as $\frac{(2m_1x_0+m_2-r)k_1}{(k_1+m_1)\Delta_p} \leq \frac{(2m_1x_0+m_2-r)k_1}{m_1\Delta_p}$.

For Proposition 5(i), note that when $\beta \in \mathbb{B}_L$, $\beta < \underline{\beta}$. In this case, we have $e^* = \frac{\bar{p}(\beta)-k_2}{2k_1}$, which we can use to simplify the derivative of $\Pi^*(\beta)$ as

$$\frac{d\Pi^*(\beta)}{d\beta} = \frac{de^{br}}{d\beta}[2m_1x_0 + m_2 - r - 2m_1e^*] - \bar{p}'(\beta)[x_0 - 1 - e^*].$$

As $e^{fb} = e^*$ holds when $\beta = \underline{\beta}$, we have $2m_1x_0 + m_2 - r - 2m_1e^* = 0$, which yields $\frac{d\Pi^*(\beta)}{d\beta} = \frac{d\Pi^{fb}(\beta)}{d\beta}$. Furthermore, we have

$$\frac{d}{d\beta}\left[\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta}\right] = \frac{(\bar{p}'(\beta))^2}{2k_1^2}(k_1 - m_1) - \frac{(\bar{p}'(\beta))^2}{2(k_1 + m_1)} = -\frac{(\bar{p}'(\beta))^2}{2k_1^2(k_1 + m_1)}m_1^2 \leq 0,$$

which implies $\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta}$ is decreasing in β. Combining this with $\frac{d\Pi^*(\beta)}{d\beta} = \frac{d\Pi^{fb}(\beta)}{d\beta}$ when $\beta = \underline{\beta}$, we have $\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta} \geq 0$ for all $\beta < \underline{\beta}$.

For Proposition 5(ii), note that $\Delta(\beta)$ is not monotone when $\beta \in \mathbb{B}_M$.

For Proposition 5(iii), note that when $\beta \in \mathbb{B}_H$, we have $\beta > \overline{\beta}$. In this case, $e^{fb} = (1 + \frac{k_1}{k_1+m_1})e^*$ and $e^* = \frac{2m_1x_0+m_2-r+\bar{p}(\beta)-k_2}{2(2k_1+m_1)}$, which we can use to simplify the derivatives of $\Pi^{fb}(\beta)$ and $\Pi^*(\beta)$ as

$$\frac{d\Pi^{fb}(\beta)}{d\beta} = -\bar{p}'(\beta)[x_0 - 1 - (1 + \frac{k_1}{k_1 + m_1})e^*], \text{ and}$$

$$\frac{d\Pi^*(\beta)}{d\beta} = 2k_1 e^{br}\frac{de^{br}}{d\beta} - \bar{p}'(\beta)[x_0 - 1 - e^{br}] = -\bar{p}'(\beta)[x_0 - 1 - (1 + \frac{k_1}{2k_1 + m_1})e^*].$$

Taking the difference yields $\frac{d\Pi^*(\beta)}{d\beta} - \frac{d\Pi^{fb}(\beta)}{d\beta} = -\bar{p}'(\beta)\left[\frac{k_1}{k_1+m_1} - \frac{k_1}{2k_1+m_1}\right] \geq 0$ for all $\beta > \overline{\beta}$, as we have $\bar{p}'(\beta) = p_s - p_v < 0$.

A.8. Proof of Corollary 2

To study the comparative statics of $\underline{\beta}$ and $\overline{\beta}$ with respect to m_1, we will only consider the case with $2m_1x_0 + m_2 - r > 0$, as $\underline{\beta}$ and $\overline{\beta}$ depend on m_1 only when the aforementioned condition is satisfied. Taking the 1st order derivatives of $\underline{\beta}$ and $\overline{\beta}$ with respect to m_1 yields

$$\frac{d\underline{\beta}}{dm_1} = \frac{k_1(m_2 - r)}{{m_1}^2\Delta_p}, \text{ and } \frac{d\overline{\beta}}{dm_1} = \frac{k_1(m_2 - r - 2k_1x_0)}{(k_1 + m_1)^2\Delta_p}.$$

Assuming $\underline{\beta}$ and $\overline{\beta}$ are positive, we use $e^* = (p_v - k_2)/(2k_1)$ when $\beta = 0$, $e^* \leq X_0 - 1$, and $p_v - k_2 \leq 2k_1(X_0 - 1)$ to establish bounds

$$0 \leq \underline{\beta} \leq \frac{k_1(-2m_1 - m_2 + r)}{m_1\Delta_p}, \text{ and}$$
$$0 \leq \overline{\beta} \leq \frac{k_1(-2m_1 - 2k_1 - m_2 + r + 2k_1x_0)}{(k_1 + m_1)\Delta_p}.$$

Here, the first expression ensures $m_2 \leq r$, whereas the second ensures $m_2 \leq r + 2k_1x_0$. Consequently, we have $\frac{d\underline{\beta}}{dm_1} \leq 0$ and $\frac{d\overline{\beta}}{dm_1} \leq 0$.

Monotonicity of $\underline{\beta}$ and $\overline{\beta}$ with respect to m_2 can be established trivially, and thus we do not present a formal proof.

B. Proofs in Section 5

We omit the proofs for Propositions 6 and 7, as they are almost identical to the proofs of Propositions 1 and 2.

B.1. Supplementary Results for Proposition 8

LEMMA 2. *Let (w_z^*, α_z^*) be the optimal contracts the buyer offers to a type $z \in \{\underline{z}, 1\}$ supplier for a given recycling rate β. Then, the following statements hold true:*

(i) $\min_z w_z^* = 0$, i.e., the buyer offers a positive fixed payment to at most one type of supplier.

(ii) $\alpha_{z_1}^* \geq \alpha_{z_2}^*$ for any $z_1, z_2 \in \{\underline{z}, 1\}$ if and only if $w_{z_2}^* \geq w_{z_1}^* = 0$, i.e., the buyer does not offer a fixed payment to the supplier type that gets a higher compensation rate.

(iii) If $\alpha_{z_1}^* \geq \alpha_{z_2}^*$ for any $z_1, z_2 \in \{\underline{z}, 1\}$, then the IC constraint for a type z_2 supplier must bind.

(iv) If $\alpha_{z_1}^* > \alpha_{z_2}^*$ for any $z_1, z_2 \in \{\underline{z}, 1\}$, then a type z_2 supplier exerts its first-best effort.

Proof of Lemma 2 : We prove Lemma 2(i) by contradiction. Assume $\min_z w_z^* > 0$ holds at optimality, while keeping the compensation rates for each type unchanged. Consider an alternative contract where the fixed payment is $w_z^* - \varepsilon$ for type $z \in \{\underline{z}, 1\}$. Note that this change does not affect the supplier's equilibrium decision, and thus both IC constraints continue to hold. As the buyer pays lower fixed payments in this alternative contract, its profits improve, which contradicts the optimality of the first contract with $\min_z w_z^* > 0$. Thus, at equilibrium we must have $\min_z w_z^* = 0$.

To prove Lemma 2(ii), we let $\pi_z(\alpha, w)$ be the optimal profit of the type $z \in \{\underline{z}, 1\}$ supplier under a contract with fixed payment w and compensation rate α, i.e., $\pi_z(\alpha, w) \equiv \max_e \mathbb{E}[\hat{\Pi}_s(X, e, z) + \hat{t}(X, w, \alpha)]$. Using the IC constraint of type z_2, we have

$$\begin{aligned}\pi_{z_2}(\alpha^*_{z_2}, w^*_{z_2}) - \pi_{z_2}(\alpha^*_{z_1}, w^*_{z_1}) \geq 0 &\Leftrightarrow w^*_{z_2} - w^*_{z_1} + \pi_{z_2}(\alpha^*_{z_2}, 0) - \pi_{z_2}(\alpha^*_{z_1}, 0) \geq 0 \\ &\Leftrightarrow w^*_{z_2} - w^*_{z_1} \geq \pi_{z_2}(\alpha^*_{z_1}, 0) - \pi_{z_2}(\alpha^*_{z_2}, 0) \geq 0,\end{aligned}$$

where the first if-and-only-if statement holds as $\pi_z(\alpha, w) = w + \pi_z(\alpha, 0)$, and the second one holds as $\pi_z(\alpha, w)$ is increasing in α (implied by the "Envelope Theorem"). Finally, $w^*_{z_1} = 0$ as per Lemma 2(i).

To prove Lemma 2(iii), first note that $w^*_{z_2} \geq w^*_{z_1} = 0$ as per Lemma 2(ii). When $w^*_{z_2} = 0$ also holds, we must have $\alpha^*_{z_1} = \alpha^*_{z_2}$ as per Lemma 2(ii), which implies that the IC constraint of type z_2 must bind. When $w^*_{z_2} > 0$, we prove by contradiction. Suppose the IC constraint of type z_2 does not bind at equilibrium, and consider an alternative contract where the fixed payment for type z_2 is $w^*_{z_2} - \varepsilon$ while keeping other contract parameters unchanged. The IC constraint for type z_1 continues to hold under this alternative contract because deviation for type z_1 becomes more difficult. Furthermore, the alternative contract improves the buyer's profit due to a lower fixed payment for type z_2, which contradicts the optimality of $w^*_{z_2}$ in the prior contract. Therefore, the IC constraint of type z_2 must bind at equilibrium.

To prove Lemma 2(iv), we define $\mathcal{C}$ as the set of contract parameters satisfying $\alpha_{z_1} > \alpha_{z_2}$, $w_{z_2} > w_{z_1} = 0$. As per Lemma 2(iii), we also assume that the IC constraint of type z_2 binds. Then, we have

$$w_{z_2} = \pi_{z_2}(\alpha_{z_1}, 0) - \pi_{z_2}(\alpha_{z_2}, 0) = \int_{\alpha_{z_2}}^{\alpha_{z_1}} e^{br}_{z_2}(\alpha) d\alpha,$$

where $\pi_z(\alpha, w) \equiv \max_e \mathbb{E}[\Pi^z_s(X, e, z) + t_z(X, w, \alpha)]$. Here, the last equality follows from $\frac{d\pi_z(\alpha,w)}{d\alpha} = e^{br}_{z_2}(\alpha, \beta)$ and the "Fundamental Theorem of Calculus."

Furthermore, as per Lemma 2(ii) and 2(iii), the optimal contract must belong in $\mathcal{C}$, and thus it should solve

$$\max_{(\alpha_{z_1}, \alpha_{z_2}, w_{z_1}, w_{z_2}) \in \mathcal{C}} V_b + \sum_{z \in \{z_1, z_2\}} \gamma_z \big[r[x_o - e^{br}_z - 1] - M(x_0 - e^{br}_z) - \alpha_z e^{br}_z - w_z \big],$$

where we denote $e^{br}_z(\alpha_z)$ by e^{br}_z for brevity. Then, the first-order condition for the objective of the aforementioned optimization problem with respect to α_{z_2} yields

$$\frac{de^{br}_{z_2}}{d\alpha_{z_2}}\left[2m_1[x_0-e^{br}_{z_2}]+m_2-r-\alpha^*_{z_2}\right]=0.$$

Then, using the optimality condition of a type z_2 supplier's problem, we have

$$2m_1[x_0-e^{br}_{z_2}]+m_2-r-2z_2k_1e^{br}_{z_2}-z_2k_2+\bar{p}(\beta)=0,$$

which coincides with the optimality condition of the first-best for a type z_2 supplier; see equation (10). Consequently, $e^{br}_{z_2}(\alpha^*_{z_2})=e^{fb}_{z_2}$ must hold.

B.2. Proof of Proposition 8

We will prove Proposition 8(i) by contradiction. Assume $\alpha^*_{\underline{z}}>\alpha^*_1$ holds optimally, and note that $e^{br}_1=e^{fb}_1$ as per Lemma 2. We also know that $e^{br}_{\underline{z}}\le e^{fb}_{\underline{z}}$ holds, because the optimality of $\alpha^*_{\underline{z}}$ implies $2m_1[x_0-e^{br}_{\underline{z}}]+m_2-r-4\underline{z}k_1e^{br}_{\underline{z}}-zk_2+\bar{p}(\beta)\ge 0$. (The last argument holds following arguments those we invoked in the Proof of Proposition 4. We define $\hat{\alpha}_z$ as the compensation rate inducing a type $z\in[\underline{z},1]$ supplier to exert its first-best effort. Then, finding the solution of $e^{br}_z(\hat{\alpha}_z)=\hat{e}^{fb}_z$ yields we have that

$$\hat{\alpha}_z=zk_2-\bar{p}(\beta)+\frac{zk_1}{m_1+zk_1}[2m_1x_0+m_2-r-zk_2+\bar{p}(\beta)],$$

which implies $\hat{\alpha}_z$ is increasing in z. Using the monotonicity of $\hat{\alpha}_z$, we can conclude that $\alpha^*_{\underline{z}}\le\alpha_{\underline{z}}\le\alpha_1=\hat{\alpha}^*_1$, where the first inequality follows from $e^{br}_{\underline{z}}\le e^{fb}_{\underline{z}}$, and the last equality follows from $e^{br}_1=e^{fb}_1$. Evidently, this contradicts $\alpha^*_{\underline{z}}>\alpha^*_1$, which implies $\alpha^*_{\underline{z}}\le\alpha^*_1$ must hold true at equilibrium. To prove $\omega^*_{\underline{z}}\ge\omega^*_1=0$, we invoke Lemma 2(ii).

Proposition 8(ii) follows from Lemma 2(ii).

To prove Proposition 8(iii), note that $e^{br}_z(\alpha_z)=\left(\frac{\alpha_z-zk_2+\bar{p}(\beta)}{2zk_1}\right)^+$, implying e^{br}_z decreases as z increases. As a result, $e^{br}_{\underline{z}}\ge e^{br}_1$ must hold (when both are positive) if a pooling equilibrium sustains, as such an equilibrium implies the same optimal compensation rate for each supplier type.

When a separating equilibrium results, note that we have $e^{br}_{\underline{z}}(\alpha^*_{\underline{z}})=e^{fb}_{\underline{z}}$ as per Lemma 2(iv). Also note that $e^{br}_1(\alpha^*_1)\le e^{fb}_1$, which follows from Proposition 4. Furthermore, as $e^{fb}_z=\frac{2m_1x_0+m_2-r-zk_2+\bar{p}(\beta)}{2(m_1+zk_1)}$, firs-best effort e^{fb}_z decreases as z increases. Therefore, we should have $e^{br}_{\underline{z}}(\alpha^*_{\underline{z}})\ge e^{br}_1(\alpha^*_1)$ at equilibrium.

To prove Proposition 8(iv), note that we can express $e^{br}_z(\alpha)$ for a type $z\in\{\underline{z},1\}$ supplier as

$$\hat{e}^{br}_z(\alpha)=\left(\frac{\alpha+\bar{p}(\beta)-zk_2}{2zk_1}\right)^+$$

when $m_1=0$. Furthermore, an optimal contract implies $\alpha_{\underline{z}}\le\alpha_1$ and $w_1=0$ as per Proposition 8(i), and the *IC* constraint of a type $\underline{z}$ supplier must bind as per Lemma 2, where the latter yields

$$w_{\underline{z}}=\int_{\alpha_{\underline{z}}}^{\alpha_1}e^{br}_{\underline{z}}(\alpha)=(\alpha_1-\alpha_{\underline{z}})\frac{e^{br}_{\underline{z}}(\alpha_{\underline{z}})+e^{br}_{\underline{z}}(\alpha_1)}{2}.$$

Then, we can express the buyer's profit, which we denote by $\pi_b(\alpha_{\underline{z}}, \alpha_1)$ as

$$\pi_b(\alpha_{\underline{z}}, \alpha_1) = V_b + (r - m_2)x_0 - r + \gamma_{\underline{z}}\left[e_{\underline{z}}^{br}(\alpha_{\underline{z}})(m_2 - r - \alpha_{\underline{z}}) - (\alpha_1 - \alpha_{\underline{z}})\frac{e_{\underline{z}}^{br}(\alpha_{\underline{z}}) + e_{\underline{z}}^{br}(\alpha_1)}{2}\right] + \gamma_1\left[e_1^{br}(\alpha_1)(m_2 - r - \alpha_1)\right]. \tag{15}$$

In this case, the buyer's optimization requires maximizing $\max \pi_b(\alpha_{\underline{z}}, \alpha_1)$ subject to $\alpha_{\underline{z}} \le \alpha_1$. Taking the derivative of $\pi_b(\alpha_{\underline{z}}, \alpha_1)$ with respect to $\alpha_{\underline{z}}$ yields

$$\frac{\partial \pi_b(\alpha_{\underline{z}}, \alpha_1)}{\partial \alpha_{\underline{z}}} = \gamma_z \frac{e_{\underline{z}}^{br}(\alpha_1)}{2}$$

for any $\alpha_{\underline{z}} \le zk_2 - \bar{p}(\beta)$, as $e_{\underline{z}}^{br}(\alpha_{\underline{z}}) = 0$ when $\alpha_{\underline{z}} \le zk_2 - \bar{p}(\beta)$ holds. This implies that $\pi_b(\alpha_{\underline{z}}, \alpha_1)$ increases as $\alpha_{\underline{z}}$ increases, and thus it can never optimal to set $\alpha_{\underline{z}}$ to a value less than $zk_2 - \bar{p}(\beta)$. Furthermore, we have

$$\frac{\partial \pi_b(\alpha_{\underline{z}}, \alpha_1)}{\partial \alpha_{\underline{z}}} = \gamma_{\underline{z}} \frac{\partial e_{\underline{z}}^{br}(\alpha_{\underline{z}})}{\partial \alpha_{\underline{z}}}\left[m_2 - r - \frac{\alpha_1 + \alpha_{\underline{z}}}{2}\right] - \gamma_z \frac{e_{\underline{z}}^{br}(\alpha_{\underline{z}}) - e_{\underline{z}}^{br}(\alpha_1)}{2} = \gamma_{\underline{z}} \frac{m_2 - r - \alpha_{\underline{z}}}{2zk_1},$$

for any $\alpha_{\underline{z}} > zk_2 - \bar{p}(\beta)$, which implies that $\pi_b(\alpha_{\underline{z}}, \alpha_1)$ increases as $\alpha_{\underline{z}}$ increases when $\alpha_{\underline{z}} < \bar{\alpha} \equiv \max\{zk_2 - \bar{p}(\beta), m_2 - r\}$ holds; and decreases otherwise, regardless of what the value of α_1 is. Therefore, either $\alpha_{\underline{z}}^* = \bar{\alpha}$, or $\alpha_{\underline{z}}^* = \alpha_1$ must hold when $\alpha_1 < \bar{\alpha}$ is satisfied. In other words, $\alpha_{\underline{z}}^*$ must lie on the line satisfying $\alpha_{\underline{z}} = \min\{\alpha_1, \bar{\alpha}\}$, which we illustrate in Figure 6. As a result, we can express the buyer's optimization as finding $\hat{\alpha}_1^*$ that maximizes $\pi_b(\min\{\alpha_1, \bar{\alpha}\}, \alpha_1)$.

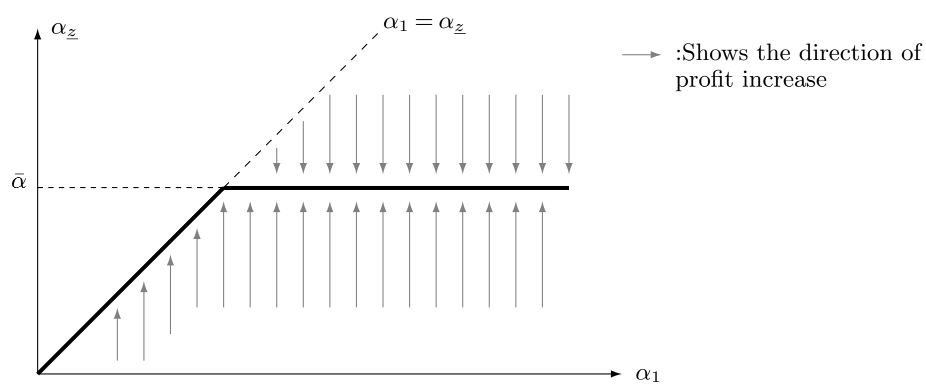

Figure 6 Illustration of the line where the optimal contract parameters must lie.

As $\hat{\alpha}_1^* \le \bar{\alpha}$ must hold under a pooling equilibrium, $\hat{\alpha}_1^* > \bar{\alpha}$ should be satisfied if it is possible for a separating equilibrium to sustain. If that were the case, then we should also have $\pi_b(\bar{\alpha}, \alpha_1) > \pi_b(\alpha_1, \alpha_1)$ for any $\alpha_1 \le \bar{\alpha}$. However, taking the derivative of $\pi_b(\bar{\alpha}, \alpha_1)$ with respect to α_1 for any $\alpha_1 > k_2 - \bar{p}(\beta)$ yields

$$\frac{d\pi_b(\bar{\alpha}, \alpha_1)}{d\alpha_1} = -\frac{\gamma_z}{2}\left[e_{\underline{z}}^{br}(\bar{\alpha}) + e_{\underline{z}}^{br}(\alpha_1) + \frac{\alpha_1 - \bar{\alpha}}{2\underline{z}k_1}\right]$$

$$+ (1-\gamma_z)\left[\frac{m_2-r-\alpha_1}{2k_1} - e_1^{br}(\alpha_1)\right]$$
$$= -\frac{\gamma_z}{2zk_1}\left[\alpha_1+\bar{p}(\beta)-zk_2\right] + \frac{1-\gamma_z}{2k_1}\left[m_2-r+k_2-\bar{p}(\beta)-2\alpha_1\right],$$

and assessing the same derivative for $\alpha_1 \le k_2-\bar{p}(\beta)$ yields

$$\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_1} = -\frac{\gamma_z}{2zk_1}\left[\alpha_1+\bar{p}(\beta)-zk_2\right],$$

as $e_1^{br}(\alpha_1)=0$ when $\alpha_1 \le k_2-\bar{p}(\beta)$. Furthermore, as the second derivative of $\pi_b(\bar{\alpha},\alpha_1)$ is negative, function $\pi_b(\bar{\alpha},\alpha_1)$ must be concave in α_1. As such proving $\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_{\underline{z}}} \le 0$ at $\alpha_1=\bar{\alpha}$ suffices to rule out a separating equilibrium, because $\pi_b(\bar{\alpha},\alpha_1)$ is decreasing in α_1 for all $\alpha_1 \ge \bar{\alpha}$ due to the concavity.

To show that $\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_{\underline{z}}}\Big|_{\alpha_1=\bar{\alpha}} \le 0$, we first consider the case with $\bar{\alpha} \le k_2-\bar{p}(\beta)$, in which case, we have

$$\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_1}\Big|_{\alpha_1=\bar{\alpha}} = -\frac{\gamma_z}{2zk_1}\left[\bar{\alpha}+\bar{p}(\beta)-zk_2\right] \le 0,$$

as $\bar{\alpha}=\max\{m_2-r, zk_2-\bar{p}(\beta)\}$. When, on the other hand, $\bar{\alpha} > k_2-\bar{p}(\beta)$ holds, we have

$$\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_1}\Big|_{\alpha_1=\bar{\alpha}} = -\frac{\gamma_z}{2zk_1}\left[\bar{\alpha}+\bar{p}(\beta)-zk_2\right] - \frac{1-\gamma_z}{2k_1}\left[2\bar{\alpha}-m_2+r+\bar{p}(\beta)-k_2\right] \le 0.$$

Consequently, as $\frac{d\pi_b(\bar{\alpha},\alpha_1)}{d\alpha_1}\Big|_{\alpha_1=\bar{\alpha}}$ always hold, a separating equilibrium cannot sustain.

B.3. Proof of Proposition 9

Recall from Proposition 8(iv) that only a pooling equilibrium can sustain when function $M(x)$ is is linear. We also show in the proof of Proposition 8(iv), the buyer's optimal contract must solve $\max_{\alpha\le\bar{\alpha}} \pi_b(\alpha,\alpha)$ where $\bar{\alpha}=\max\{m_2-r, \underline{z}k_2-\bar{p}(\beta)\}$ and $\pi_b(\alpha_{\underline{z}},\alpha_1)$ is the buyer's profit as prescribed in (15). Using the concavity of $\pi_b(\alpha,\alpha)$ in α, we can express the first-best and equilibrium effort levels for a type $z\in\{\underline{z},1\}$ supplier as

$$e_z^{fb} = \left(\frac{m_2-r+\bar{p}(\beta)-zk_2}{2zk_1}\right)^+ \text{ and } e_z^{br}(\alpha_z) = \left(\frac{\alpha_z+\bar{p}(\beta)-zk_2}{2zk_1}\right)^+.$$

To prove Proposition 9(i), also note that $m_2-r \le \underline{z}k_2-\bar{p}(\beta)$ when $e_{\underline{z}}^{fb}(\beta)=0$. In this case, both $e_1^{fb}(\beta)=0$ and $\bar{\alpha}=\underline{z}k_2-\bar{p}(\beta)$ must also hold. Then, for any $\alpha\le\bar{\alpha}$, we must have $e_z^{br}(\alpha_z)=0$ for all $z\in\{\underline{z},1\}$. As both the first-best and the equilibrium effort levels are zero for each supplier type, we conclude that decentralization cost $\Delta(\beta)$ must be zero.

To prove Proposition 9(ii), we first consider the case where $e_1^{fb}(\beta)=0$, which is true when $m_2-r\le k_2-\bar{p}(\beta)$ holds. In this case, an inefficient supplier does not exert any effort at equilibrium, i.e., $e_1^{br}(\alpha_1)=0$, for any $\alpha\le\bar{\alpha}$. As a result, the buyer's optimization matches its counterpart in the symmetric information case with only one possible supplier type having a forging size reduction cost equal to $\underline{z}K(\cdot)$. Then, as per Proposition 5, decentralization cost $\Delta(\beta)$ must be positive as $e_{\underline{z}}^{fb}(\beta)>0$, and is decreasing in β.

Considering the case with $m_2 - r > k_2 - \bar{p}(\beta)$, which implies $e_1^{fb}(\beta) > 0$, both supplier types would exert their first-best efforts only when $\alpha = m_2 - r$ holds. However, the buyer's profit decreases as α increases when $\alpha = m_2 - r$, i.e.,

$$\frac{d\pi_b(\alpha,\alpha)}{d\alpha}\Big|_{\alpha=m_2-r} = \frac{\gamma_{\underline{z}}}{2\underline{z}k_1}\left[\underline{z}k_2 - \bar{p}(\beta) - m_2 + r\right] + \frac{1-\gamma_{\underline{z}}}{2k_1}\left[k_2 - \bar{p}(\beta) - m_2 + r\right] < 0.$$

Thus, $\alpha^* < m_2 - r$ must hold at equilibrium, which implies $e_1^{fb}(\beta) > 0$, and yields $\Delta(\beta) > 0$.